Modeling Dependability Measures

Ali Mili, Alex Thomasian, Alex Vinokurov

College of Computer Science
New Jersey Institute of Technology
Newark NJ 07102-1982 mili@cis.njit.edu

Frederick T. Sheldon
U.S. DOE Oak Ridge National Lab
PO Box 2008, MS 6085, 1 Bethel Valley Rd
Oak Ridge TN 37831-6085 sheldonft@ornl.gov

Rahma Ben Ayed Lamia Labeled Jilani
ENIT, University of Tunis Institut Superieur de Gestion
Belvedere, 1002 Tunisia Bardo 2000 Tunisia
rahma.benayed@enit.rnu.tn Lamia.Labeled@isg.rnu.tn

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Abstract

In past work, we had discussed a uniform model to represent different verification techniques, and have shown how this model can be used to support two divide-and-conquer strategies: How to compose eclectic verification claims; and how to decompose composite verification goals. In this paper, we broaden the original model, most notably by integrating cost considerations, and by encompassing multiple dimensions of dependability (reliability, security, safety). We briefly illustrate our approach with a very simple demo, that we run on a very elementary, tentative prototype. This paper does not offer conclusive research results as much as it offers motivated premises, concepts and approaches for further research.

Keywords

Dependability, reliability, safety, security, verification, testing, fault tolerance.

1 Introduction: An Eclectic Approach to Dependability

In [13], we had presented a refinement based framework in which we cast many distinct verification methods. Among the contributions of this work, we cite:

- A Discipline for Composing Verification Claims. A question that the framework proposed in [13] is intended to address is the following: Given a software product \( P \) that we have verified (using static analysis techniques) against some specification \( V \),
that we have tested against some oracle $\Omega$ using some test data $D$ and that we have made fault tolerant using some run time checks and some recovery routines, what can we claim about the correctness of $P$? How can we compose the individual verification claims into a single cumulative verification result?

- \textit{A Discipline for Decomposing Verification Goals.} The framework proposed in [13] is also intended to address the following (dual) question: Given that we have to verify product $P$ against a compound specification $R$, how can we decompose $R$ so that if we independently verify the correctness of $P$ against the various components of $R$ (possibly using different methods), we can infer that $P$ is correct with respect to $R$?

- \textit{A Discipline for Controlling Verification Costs.} In the perennial debate on the relative merits of static analysis versus testing versus fault tolerance, one simple observation seems to have been lost: most of the time, what makes a method difficult to apply is not the method itself, but the specification against which this method is applied. In [13] we have characterized specifications that are good candidates for each type of verification method.

In this paper we extend out previous work in three orthogonal directions:

- First, by replacing the logical claims of the original model with probabilistic claims, on the grounds that all methods, even formal methods, produce claims with associated degrees of (un)certainty, and with associated implicit conditions, which probability theory is equipped to capture and reason about.
• Second, by expanding the model to include, not only reliability claims, but also claims dealing with safety and security, on the grounds that these claims are interdependent, and that from the user’s standpoint it does not matter whether the failure of a system is due to faulty design or to malicious activity.

• Third, by integrating failure cost into the equation, on the grounds that a complex specification typically has many components, whose failures carry widely varying costs, that we must account for in a differential manner.

In section 2 we briefly introduce the background of our study, and highlight the main findings of [13] as they pertain to our study. In section 3 we introduce the proposed new model, and discuss how it can be used to support the management of dependability claims and goals. In section 4 we complement the discussions of section 3 by showing how we propose to capture safety claims and security claims, in addition to traditional reliability claims. In section 5 we briefly present a tool that supports the model proposed in section 3 and give a simple demo that illustrates our vision of how it can be used to support dependability management. Finally, we briefly summarize and assess our findings in section 6.

This paper does not produce results in the sense of solutions that are analyzed, validated and deployed; rather, it offers motivated ideas and proposals, that serve as a launching pad for further research. To enhance readability, the discussion in this paper will remain relatively non-technical, referring the interested reader to other references as needed.
2 Background: Genesis of our Approach

In this section we will briefly present the main contributions of [13], then we discuss how and why we propose to extend this work, thereby laying the groundwork for our subsequent developments. For the sake of readability, we will keep the discussion of this section (and most of the paper, in fact) fairly non-technical, referring interested readers to bibliographic sources for details; though we may sometimes present mathematical formulas, to fix the reader’s ideas, we do not consider that understanding the details of these formulas is required to follow our discussions. Also, while the formulas we present refer to a relation-based refinement calculus, we submit that most of our claims hold for most specification/refinement models. Indeed, it is possible to define the concept of refinement in any specification model, and to build our arguments from the ground up using the model-specific refinement ordering.

2.1 Refinement Calculi

Without significant loss of generality, we use homogeneous relations (i.e. relations from some set $S$ to itself) to represent functional specifications and program functions. Among constant relations on some space $S$ we consider the universal relation $(S \times S)$, that we denote by $L$; the identity relation $((s, s) | s \in S)$, that we denote by $I$, and the empty relation $\emptyset$, that we denote by $\phi$. We denote the relational product by mere concatenation, i.e. $RR'$ as the product of $R$ by $R'$, and we use the hat symbol ($\hat{R}$) to represent relational inversion. A relation $R$ is said to be total if and only if $I \subseteq RR'$; and a relation is said to be deterministic
if and only if $\tilde{R}R \subseteq I$.

We introduce the refinement ordering between relations (interpreted as specifications) as follows: $R$ refines $R'$ if and only if

$$RL \cap R' L \cap (R \cup R') = R'.$$

We denote this property by $R \supseteq R'$ or $R' \subseteq R$ and we admit that this is a partial ordering (i.e. it is reflexive, antisymmetric and transitive). Intuitively, $R$ refines $R'$ if and only if $R$ captures all the requirements information of $R'$. Also, a program $P$ is correct with respect to a specification $R$ if and only if the program’s function refines $R$. To further convey the meaning of the refinement ordering, we note that $R$ refines $R'$ if and only if any program that is correct with respect to $R$ is (a fortiori) correct with respect to $R'$.

In addition to its ordering properties, the refinement relation also has lattice-like properties [3]. Two specifications $R$ and $R'$ are said to be consistent if they can be refined simultaneously; in relational terms, this is written as:

$$RL \cap R' L = (R \cap R')L.$$

To briefly illustrate this condition, consider that $R$ and $R'$ are specifying the final values of some variable $x$; if $R$ and $R'$ impose, respectively, the following conditions

$$6 \leq x \leq 9,$$
then $R$ and $R'$ are consistent, because we can satisfy them simultaneously by taking

$$6 \leq x \leq 7.$$ 

But if, instead, $R$ and $R'$ imposed the following conditions

$$6 \leq x \leq 9,$$

$$12 \leq x \leq 18,$$

then $R$ and $R'$ would not be consistent, since no value of $x$ satisfies both of these conditions. The reason why consistency is important for the purpose of lattice properties is that only consistent relations admit a join (least upper bound). If $R$ and $R'$ are consistent then they admit a join with respect to the refinement ordering, which is denoted by $R \sqcup R'$ and defined by

$$R \sqcup R' = \overline{R \land R'} \cup \overline{R \land R} \cup R \sqcap R' \sqcap R.$$

The join captures all the requirements information in $R$ and all the requirements information in $R'$ (upper bound) and nothing more (least). The join can further be characterized by the following premise (given that $R$ and $R'$ are consistent):

$$Q \sqsubseteq R \land Q \sqsubseteq R' \iff Q \sqsubseteq (R \sqcup R').$$
Whereas the join is conditional, the meet is not: any two relations $R$ and $R'$ have a meet (greatest lower bound), which is denoted by $R \cap R'$ and defined by:

$$R \cap R' = RL \cap R'L \cap (R \cup R').$$

The meet represents all the requirements information that is captured simultaneously by $R$ and $R'$. The meet can further be characterized by the following premise:

$$Q \supseteq R \lor Q \supseteq R' \Rightarrow Q \supseteq (R \cap R').$$

The set of relational specifications on some space $S$ have a universal lower bound (the empty relation) but have no universal upper bound; instead, they have many maximal elements (namely, all the total deterministic relations). The structure of this ordered set is summarily illustrated in figure 1.

### 2.2 Composing Dependability Claims

In this section we use the lattice-like structure of the refinement ordering to discuss how to compose dependability claims. Specifically, we consider a system that we have statically verified for some correctness criterion, that we have tested against some functional oracle using some test data, and that we have made fault tolerant by appropriate assertions and recovery routines, the question we wish to ask is: How can we add up the individual claims that each measure allows us? What claims do these measures, combined together, allow
Figure 1: Overall Lattice Structure
us to make? How do we know whether the measures we have taken (testing, proving, fault tolerance) are complementing each other, or whether they are testing the same aspects over and over again?

To answer these questions, we have (in [13]) proposed a common refinement based model, in which we cast all three families of methods (static verification, testing, and fault tolerance); we have shown that all three methods can be interpreted as establishing that the system being analyzed refines some specification, that depends on the method and the method’s parameters. Using the join operator, we can then compose eclectic measures, stemming from different methods, into a single claim. Specifically, we discuss below, briefly, how we interpret all three families of methods by means of refinement. We let $P$ be the program that we are interested in.

- **Static Verification.** If we prove by static analysis that $P$ is correct with respect to some specification $V$, we represent this claim by:

  $$P \trianglerighteq V.$$ 

- **Testing.** We assume that we have tested program $P$ on test data $D$ using oracle $\Omega$, and that all tests have been executed successfully (if not, we redefine $D$), we claim that this allows us to write:

  $$P \trianglerighteq D \setminus \Omega,$$

where $D \setminus \Omega$ represents the (pre) restriction of $\Omega$ to $D$. Details can be found in [13].
• **Fault Tolerance.** If we test some condition $C$ at run-time, and whenever the condition does not hold we invoke a recovery routine $W$, then we can claim that:

$$P \sqsupseteq C \cap W,$$

where we take the liberty to use the same symbol $(C)$ to represent the condition and the relation that represents it, and to use the same symbol $(W)$ to represent the recovery routine and the relation that represents it. Because we do not know for each execution whether $C$ holds or not, we do not know whether we can claim $P \sqsupseteq C$ (if $C$ holds) or $P \sqsupseteq W$ (if $C$ does not hold). Since we are assured that $P$ refines at least one of them at each execution, we know that it refines their meet.

From static analysis, we infer: $P \sqsupseteq V$. From (certification) testing, we infer: $P \sqsupseteq T$, where $T = D \setminus \Omega$. From fault tolerance, we infer: $P \sqsupseteq F$, where $F = C \cap W$. From lattice theory, we infer:

$$P \sqsupseteq (V \sqcup T \sqcup F).$$

### 2.3 Decomposing Dependability Goals

A more interesting application of the lattice of refinement involves decomposing a complex dependability goal into simpler subgoals. Imagine that we must prove that some product $P$ refines a complex specification $R$, and imagine that $R$ is structured as the join of several simpler subspecifications, say $R_1$, $R_2$, ... $R_k$; we had shown in [3] that the join offers a natural mechanism to structure complex specifications as aggregates of simpler specifications.
Lattice properties provide that in order to prove $P \sqsupset R$, it suffices to prove $P \sqsupset R_i$ for all $i$.

We consider the question: which method (among static verification, testing, fault tolerance) is best adapted for each subspecification $R_i$. This question is discussed in some detail and illustrated in [13]. We summarize it briefly here:

- **Static Verification.** Ideal candidates for static verification are relations that are reflexive and transitive. Indeed, static verification usually revolves around inductive arguments (of loops, recursive calls); the reflexivity of the specification makes the basis of induction trivial, and the transitivity of the specification makes the induction step trivial. What makes static verification very difficult in general is the need to invent or guess invariant assertions and intermediate assertions; when the specification at hand is reflexive and transitive, it can be used as a sufficient (i.e. sufficiently strong) assertion throughout the program. Verifying programs against reflexive transitive specifications is so straightforward, it can actually be readily automated.

- **Testing.** Ideal specifications for testing are relations that can be coded reliably, as we do not want a faulty oracle to mislead the whole testing process. Because testing is done off-line (in the sense: not during the normal operation of the system), execution efficiency is not a major consideration (by contrast with executable assertions), but reliability of the oracle is.

- **Fault Tolerance.** Ideal specifications for fault tolerance are unary relations, i.e. relations that refer to the current state but not to past states (for example, in a sorting
program, checking that the current array is sorted is a unary property, while check-
ing that the current array is a permutation of the initial array is a binary property).
What makes fault tolerance techniques inefficient is the need for saving past states
(memory overhead) and for checking the correctness of current states with respect to
past states (CPU overhead). With unary specifications, we are spared both of these
overheads.

These orthogonal requirements are summarized in table 2, which assigns to each candidate
specification a vector of three values that indicates how adapted the specification is with
respect to each of the three methods. Given a specification \( R_i \), we review one by one all
the criteria of the table (the columns); for each criterion, we select the vector (half column)
that corresponds to the attribute of \( R_i \). The sum of all these (five) vectors gives a 3-value
vector that reflects the adequacy of the three methods (Proving, Testing, Tolerance) for
the specification at hand (\( R_i \)).

In [13] we show an example of application where the overall verification effort of a
program with respect to a compound specification is significantly smaller than the effort
of applying any one of the methods.

3 A Unified Representation

In this section we critique the model presented in the previous section, then propose a
generalization that addresses some of its shortcomings.
<table>
<thead>
<tr>
<th>Features</th>
<th>Arity</th>
<th>Reflexivity and Transitivity</th>
<th>Coding Complexity</th>
<th>Execution Time</th>
<th>Inductive Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Methods</td>
<td>1</td>
<td>2</td>
<td>Y</td>
<td>L</td>
<td>H</td>
</tr>
<tr>
<td>Proving</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
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<tr>
<td>Testing</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>Tolerance</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

Figure 2: Assessing the Adequacy of Candidate Methods

3.1 The Need for Generalization

In order to motivate the need for generalizing the model presented in the previous section, we briefly discuss why it is inadequate, as it stands.

- Most dependability measures are best modeled as probabilistic claims rather than firm logical claims.

- Most claims are contingent upon implicit conditions. For example, testing is contingent upon the condition that the testing environment is a faithful simulation of the operating environment (or, more precisely, that it is at least as harsh as the operating environment). Also, static verification is contingent upon the condition that the verification rules used in the static proof are borne out by the compiler and the operating environment. Also, fault tolerance is contingent upon the condition that the assertion-checking code and the recovery code are free of faults.
• Many claims may lend themselves to more than one interpretation. For example, if we test $P$ against oracle $\Omega$ using test data $D$, we can interpret this in one of two ways: either that $P$ refines $D \setminus \Omega$ with probability 1.0 (subject to the hypothesis discussed above, that the testing environment subsumes the operating environment); or that $P$ refines $\Omega$ (not restricted to $D$ this time), subject to the subsumption hypothesis, and to the hypothesis that $D$ is a faithful representative of the program’s domain (i.e. $P$ fails on $D$ if and only if it fails on the whole domain), with some probability $p$ less than 1.0. While the logic, refinement based, model discussed in section 2 represents only the first interpretation, the probabilistic model can represent both. In addition, we will see how the proposed model allows us to keep both interpretations, and makes use of them both (which is only fair, since they are both plausible interpretations).

• If we admit the premise that dependability claims are probabilistic, we must now consider failure costs. It is not enough to know that $P$ refines $R_i$ with some probability $p_i$, over some period of operational time; we must also know what costs we will incur in the case (probability $(1 - p_i)$) that $P$ fails to refine $R_i$ during that time.

• While the refinement ordering proves to be adequate for representing reliability claims and safety claims, as we will discuss subsequently, it is not adequate for representing security claims. We wish to generalize the the form that specifications can take, and consequently also generalize the concept of refinement to capture security properties.
3.2 A Generalized Model

We submit the premise that dependability methods can be characterized by the following features:

- **Property.** This feature represents the property that we want to establish about \( P \):
  In section 2 we were interested exclusively in refinement, but it is possible to imagine other properties, such as performance (with respect to performance requirements), security (with respect to security requirements), recoverability preservation [14] (with respect to functional requirements), etc.

- **Reference.** This feature represents the reference with respect to which we are claiming the property cited above. This can be a functional specification (if the property is correctness, or recoverability preservation), an operational specification (if the property is a performance property), or a security specification (if the property is a security property), etc.

- **Assumption.** This is the condition assumed by the verification method; all verification methods are typically based on a set of (often) implicit assumptions, and are valid only to the extent that these assumptions hold. We propose to make these assumptions explicit, so that we can reason about them.

- **Certainty.** This feature represents the probability with which we find that the property holds about \( P \) with respect to the reference, conditional upon the Assumption. The same dependability measure (e.g. testing \( P \) with respect to some oracle using
some test data, proving a refinement property with respect to some specification, etc) can be interpreted in more than one way, possibly with different probabilities.

• **Failure Cost.** Safety and security requirements are usually associated with costs, which quantify the amount of loss that results from failing to meet them. Safety costs may include loss or endangerment of human life, financial loss, endangerment of a mission, etc. Security costs may include disclosure of classified information, loss of availability, exposure of personal information, etc. The purpose of this feature is to quantify this cost factor, and associate it explicitly with the failure that has caused it.

• **Verification Cost.** Verification costs complement the information provided by failure costs, by quantifying how much it costs to avoid failure, or reduce the probability of failure. Together these two functions help manage risks and risk mitigation. The table given in Figure 2 represents a first draft of verification costs, since it represents, be it in relative terms, the cost associated with deploying each method against a given specification.

To reflect this characterization, we represent dependability claims as follows:

\[ \Pi(P \sqsupseteq R|A) = p, \]

where \( P \) is the product, \( \sqsupseteq \) is the property we claim about it, \( R \) is the specification against which we are making the claim, \( A \) is the assumption under which we are making the claim,
and $p$ is the probability with which we are making the claim. We further add two cost functions:

- **Failure Cost**: This function (which we denote by $\phi$) maps a property (say, $\sqsubseteq$) and a reference (say, some specification $R$) into a cost value (quantified in financial terms, or in terms of human lives at risk, etc). Hence

$$\phi(\sqsubseteq, R)$$

represents the cost that we expect to incur whenever $P$ fails to satisfy property $\sqsubseteq$ with respect to $R$.

- **Verification Cost**: This function (which we denote by $\nu$) maps a property (say, $\sqsubseteq$), a reference (say, $R$), an assumption (say, $A$) and a method (say, $M$), to a cost value (expressed in Person Months). Hence

$$\nu(\sqsubseteq, R, A, M)$$

represents the cost of applying method $M$ to prove that $P$ satisfies property $\sqsubseteq$ with respect to $R$ under the assumption $A$.

Although this model appears on the face of it to deal only with claims that pertain to the whole system $P$, we can in fact use it to represent verification steps taken on components of $P$ [19]. We use an illustrative example: We let $P$ be the composition of two components, say $P_1$ and $P_2$, and we assume that we have used some method to establish the following
claim:

\[ \Pi(P_1 \supseteq R_1 | A) = p. \]

We submit that this can be written as a property of \( P \) (rather than merely a property of \( P_1 \)) if we add an assumption about \( P_2 \). Hence, for example, we can infer a claim of the form

\[ \Pi(P \supseteq (R_1 R_2) | A \land P_2 \supseteq R_2) = p', \]

for some reference \( R_2 \) and some probability \( p' \). We submit that this model enables us to collect every piece of information that we can derive from dependability measures, so that all the verification effort that is expended on \( P \) can be exploited (to support queries, as we will discuss in section 5).

### 3.3 Implications of the Model

The first implication of this probabilistic model is that verification claims are no longer additive, in the sense that we discussed in section 2. While in the logic, refinement-based model we could sum up all our claims in a single refinement property, in the new probabilistic model it is generally not possible to do so. Nor is it desirable, in fact, as the result would probably be so complex as to be of little use. What we advocate instead is to use an inference system where all the collected claims can be stored, and subsequently used to answer queries about the dependability of the system. This will be illustrated in section 5 through a simple example.

The second implication of this model is that it allows us to introduce a measure of
dependability that integrates cost information. When we say that a system $P$ has a given MTTF, it is with respect to some implicit specification, say $R$. It is also with respect to some implicit understanding of failure cost, i.e. how much we stand to lose if our system fails to satisfy $R$. If we consider that $R$ is an aggregate of several subspecifications, say $R_1$, $R_2$, ... $R_k$, it is conceivable that the components $R_1$, $R_2$, ... $R_k$ have different failure costs associated with them; for example, failing to refine $R_1$ will cost significantly more than failing to refine $R_k$, but the MTTF does not reflect this, as it considers both as failures to refine $R$. We introduce the concept of Mean Failure Cost (MFC), which combines terms of the form

$$\Pi(P \sqsupseteq R_i) \times \phi(\sqsupseteq, R_i)$$

where the term $\Pi(P \sqsupseteq R_i)$ represents the probability that $P$ fails to refine $R_i$ and $\phi(\sqsupseteq, R_i)$ represents the cost that we incur when it does.

4 Capturing Dependability Measures

In this section we discuss the representation of dependability claims using the model presented above, and extensions of it.
4.1 Reliability and Safety

We consider a specification \( R \) and a software product \( P \) that is developed to satisfy \( R \); we assume that \( R \) is decomposed as the join of several subspecifications, i.e.

\[
R = R_1 \sqcup R_2 \sqcup \ldots \sqcup R_k.
\]

Also, we let

\[
\Gamma_1, \Gamma_2, \ldots \Gamma_k
\]

be the failure costs associated with specifications \( R_i \), i.e.

\[
\phi(\sqcup, R_i) = \Gamma_i.
\]

Further, we let \( Q \) be a safety requirement that \( P \) must also satisfy, in addition to satisfying \( R \), and we let \( \Gamma' \) be the failure cost associated with \( Q \), i.e.

\[
\phi(\sqcup, Q) = \Gamma'.
\]

Program \( P \) is considered reliable if it refines \( R \) and safe if it refines \( Q \). But most generally, \( R \) refines \( Q \), which seems to suggest, superficially, that if \( P \) is reliable then it is safe — which is not the case, of course. What is missing from this inference is the implicit assumption that \( P \) must refine \( R \) and \( Q \) with different degrees of certainty, because they are associated with vastly different failure costs.
We argue that there is no difference between reliability and safety; both are modeled by
the property that \( P \) refines some specification, with some associated failure cost. Figure 3
illustrates the refinement relationships between reliability specifications and safety speci-
cfications. Specification \( Q \) is not necessarily comparable to the subspecifications \( R_i \), but \( R \),
which the join of the \( R_i \), refines \( Q \). The \( R_i \) may have varying failure costs associated with
them; what sets \( Q \) apart is that its failure cost is very high.

We argue that rather than quantify reliability and safety by means of two separate
MTTF’s, we can quantify both of them by means of the Mean Failure Cost (MFC), which
is a combination of terms of the form

\[
\Pi(P \sqsupset S_p) \times \phi(\sqsubseteq, S_p).
\]

The higher the term \( \phi(\sqsubseteq, S_p) \), the lower the term \( \Pi(P \sqsupset S_p) \) must be to keep the mean
failure cost low. To reduce the term \( \Pi(P \sqsupset S_p) \) we must increase the term \( \Pi(P \sqsubseteq S_p) \),
whence the need to ensure that we satisfy safety requirements with a higher probability
than reliability requirements.

4.2 Modeling Security

Even though logically, system reliability is driven exclusively by the existence and possi-
ble manifestation of faults, empirical observations regularly show a very weak correlation
between faults and reliability. In [15], Mills and Dyer discuss an example where they find
a variance of 1 to 50 in the impact of faults on reliability; i.e. some faults cause system
\[ R = R_1 \sqcup R_2 \sqcup \ldots \sqcup R_k \]

\[ \text{Prob}(\text{Not Reliable, Safe}) = 0.0099 \]

\[ \text{Prob}(\text{Not Reliable, Not Safe}) = 0.0001 \]

Figure 3: Refinement Relations, Reliability and Safety
failure 50 times more often than others; while their experiment highlights a variance of 1 to 50, we have no doubt that actual variance is in fact unbounded. Also, they find that they can remove 60 percent of a system’s faults and improve its reliability by only ... 3 percent.\textsuperscript{1} In a study of IBM software products, Adams [1] finds that many faults in the system are only likely to cause failure after hundreds of thousands of months of product usage.

We argue that the same may be true for security: vulnerabilities in a system may have widely varying impacts on system security. In fairness, the variance may be wider for reliability than for security, because in malicious security violations high impact vulnerabilities may be more attractive targets than lower impact vulnerabilities, but wide variances are still quite plausible. Wide variances, to the extent that they are borne out, have broad impacts on security management:

- In practice, security ought not be defined as the absence of vulnerabilities, no more than reliability is defined by the absence of faults (low impact vulnerabilities do not affect security in a meaningful way).

- In practice, security ought not be measured/quantified by the number of vulnerabilities, just as it is widely agreed (as highlighted by Adams’ [1] and Mills’ [15] work) that faults per KLOC is an inappropriate measure of reliability. Though fault density is commonly used as a measure of programmer productivity/product quality, it has

\textsuperscript{1}Given that typically system level testing consumes nearly 50\% of life-cycle costs and hardly comes close to discovering 60\% of system faults, this finding is a resounding condemnation of random fault-chasing, and advocates instead a discipline that leads us towards the most influential faults first.
long been discredited as a measure of reliability.

- Security cannot be improved by focusing on vulnerabilities, as we have no way to tell whether a given vulnerability has low (1) or high (50) impact on security. Rather, security should be managed by pursuing a policy that leads us to the highest impact vulnerabilities first (a similar approach to usage pattern testing [16, 15, 9, 10, 2, 8, 6, 18]).

In light of these observations, we argue in favor of modeling security in a way that reflects its visible, measurable, observable attributes, rather than its hypothesized causes. To this effect, we introduce the following features:

- A notation for security specification, which details how to capture security requirements of a system.

- A formula for security certification, which formulates the condition under which a system meets a given set of security requirements (represented by security specifications).

In [17] Nicol et al. discuss a number of dimensions of security, including: data confidentiality, data integrity, authentication, survivability, non-repudiation, etc. In the context of this paper, we focus our attention on survivability, and readily acknowledge a loss of generality; other dimensions of security are under investigation. Survivability is defined in [7] as the capability of a system to fulfill its mission in a timely manner, in the presence of attacks, failures, or accidents [17]. We discuss in turn how to represent security (surviv-
ability) requirements, and how to represent the claim that a system meets these security requirements.

4.2.1 Specifying Security Requirements

We note that there are two aspects to survivability: the ability to deliver some services, and the ability to deliver these services in a timely manner; to accommodate these, we formula security requirements by means of two relations, one for each aspect. Using a relational specification model presented in [4, 5] we propose to formulate functional requirements as follows:

- An input space, that we denote with $X$; this set contains all possible inputs that may be submitted to the system, be they legitimate or illegitimate (part of an attack/intrusion).

- Using space $X$, we define space $H$, which represents the set of sequences of elements of $X$; we refer to $H$ as the set of input histories of the specification. An element $h$ of $H$ represents an input history of the form

$$..h_N.h_{N-1}...h_3.h_2.h_1.h_0,$$

where $h_0$ represents the current input, $h_1$ represents the previous input, $h_2$ represents the input before that, etc.

- An output space $Y$, which represents all possible outputs of the system in question.
• A relation $\beta$ from $H$ to $Y$ that specifies for each input history $h$ (which may include intrusion/attack actions) which possible outputs may be considered correct (or at least acceptable). Note that $\beta$ is not necessarily deterministic, hence there may be more than one output for a given input history. Note also that this relation may be different from relation $R$ which specifies the normal functional requirements of the system: while $R$ represents the desired functional properties that we expect from the system, $\beta$ represents the minimal functional properties we must have even if we are under attack; hence while it is possible to let $\beta = R$, it is also possible (perhaps even typical) to let there be a wide gap between them.

As for representing timeliness requirements, we propose the following model:

• The same input space $X$, and history space $H$.

• A relation from $H$ to the set of positive real numbers, which represents for each input history $h$ the maximum response time we tolerate for this input sequence, even in the presence of attacks. We denote this relation by $\omega$.

In the sequel, we discuss under what condition do we consider that a system $S$ satisfies the security requirements specified by the pair $(\beta, \omega)$.

4.2.2 Certifying Security Properties

Given a security requirements specification of the form $(\beta, \omega)$, we want to discuss under what condition we consider that a program $S$ that takes inputs in $X$ and produces outputs in $Y$ can be considered to satisfy these security requirements. Space limitations preclude
us from a detailed modeling of attacks/ intrusions, hence we will, for the purposes of this paper, use the following notations:

- Given a legitimate input history $h$, we denote by $v(h)$ an input history obtained from $h$ by inserting an arbitrary intrusion sequence (i.e. sequence of actions that represent an intrusion into the system).

- Given an input history $h$ (that may include intrusion actions) we denote by $\theta(S, h)$ the response time of $S$ on input history $h$.

Using these notations, we introduce the following definition.

**Definition 1** A system $S$ is said to be secure with respect to specification $(\beta, \omega)$ if and only if

1. For all legitimate input history $h$,

   $$(h, S(h)) \in \beta \Rightarrow (v(h), S(v(h))) \in \beta.$$ 

2. For all legitimate input history $h$,

   $$\theta(S, h) < \omega(h) \Rightarrow \theta(S, v(h)) < \omega(h).$$ 

The first clause of this definition can be interpreted as follows: if system $S$ behaves correctly with respect to $\beta$ in the absence of an intrusion, then it behaves correctly with respect to $\beta$ in the presence of an intrusion. Note the conditional nature of this clause: we are not
saying that $S$ has to satisfy $\beta$ at all times, as that is a reliability condition; nor are we saying that $S$ has to satisfy $\beta$ in the presence of an intrusion, as we do not know whether it satisfies in the absence of an intrusion (surely we do not expect the intrusion to improve the behavior of the system —all we hope for is that it does not degrade it). Rather we are saying that if $S$ satisfies $\beta$ in the absence of an intrusion, then it satisfies it in the presence of an intrusion.

The second clause articulates a similar argument, pertaining to the response time: if the response time of $S$ was within the boundaries set by $\omega$ in the absence of an intrusion, then it remains within those bounds in the presence of an intrusion.

At the risk of overloading the refinement symbol ($\sqsupseteq$), we resolve to use it to represent the property that a system $P$ is secure (according to the definition above) with respect to a security specification $(\beta, \omega)$. The form of the specification, when it is explicit, resolves the ambiguity. Hence we write

$$P \sqsupseteq (\beta, \omega)$$

to mean that $P$ is secure with respect to $(\beta, \text{operational part})$.

### 4.2.3 Integrating Security

The definition that we propose here is focused entirely on effects rather than causes, and gives meaning to the concept of security failure. Using this concept, we can now quantify security by adding terms of the form

$$\Pi(P \sqsupseteq R) \times \phi(\sqsupseteq, R)$$
to the mean failure cost, producing a function that quantifies the expected failure cost, without distinction on whether the failure is due to a design fault (reliability, safety) or an intrusion (security). In [20] Stevens et al. present measures of security in terms of MTTD (D: vulnerability discovery) and MTTE (E: exploitation of discovered vulnerability). By contrast with our (re) definition, these definitions are focused on causes (rather than effect); in fairness, Stevens et al. propose them as intruder models rather than security models. The difference between our effect-based measure and Stevens’ cause-based measure is that a vulnerability may be discovered without leading to an intrusion, and an intrusion may be launched without leading to a security failure in the sense of our definition.

5 Illustration: Dependability Queries

In the previous section we discussed how we can represent dependability claims in a unified model; in this section, we briefly discuss how to deploy claims represented in this manner to support queries. We will first discuss, in broad terms, some inference rule; then we show a sample example of illustration.

5.1 Inference Rules

We envision a database in which we cumulate all the verification claims that we obtain, from various methods, applied to various components (though typically the whole system), against various specifications (functional specifications, security specifications, etc), reflecting various properties (correctness, security, recoverability preservation, etc). Queries are
submitted to this database and inference rules allow us to determine how to answer the
query in light of available claims. We classify inference rules into a number of categories:

- **Probability Rules.** This category includes all the rules that stem from probability
  theory, including especially identities that pertain to conditional probability.

- **Refinement Rules.** This category includes all the rules that stem from the partial
  order structure of the refinement ordering. For example, if $R$ refines $R'$, then we
  know, by transitivity of the refinement ordering, that

  $$\Pi(P \supseteq R|A) \leq \Pi(P \supseteq R'|A).$$

- **Lattice Rules.** This category includes all the rules that stem from the lattice structure
  of the refinement ordering. For Example, if $R_1$ and $R_2$ are specifications that admit
  a join, we have

  $$\Pi(P \supseteq (R_1 \sqcup R_2)|A) \geq \Pi(P \supseteq R_1) \times \Pi(P \supseteq R_2).$$

- **Conversion Rules.** This category includes the rules that reflect relationships between
  the various properties that we wish to claim (correctness, recoverability preservation,
  security, etc). Examples of relations that we capture by these rules include: the fact
  that if $P$ is correct with respect to $R$, it is recoverability-preserving with respect to $R$;
  the fact that the security of $P$ is contingent upon the correctness of the components
  that enforce its security policies; etc.
5.2 A Tool Prototype

We have developed a very sketchy prototype of a tool that stores claims and supports queries. In its current form, the prototype includes only probability rules, hence has very limited capability. Nevertheless, it allows us to discuss our vision of its function and its operation. The first screen of the prototype offers the following options:

- **Record a Reliability/ Safety Claim.** Clicking on this tab prepares the system for receiving details about a dependability claim (reliability, safety, etc) with respect to a functional specification. Given that such claims have the general form:

\[ \Pi(P \sqsupset R|A) = p, \]

the system prompts the user to fill in fields for the property (\( \sqsupset \)), the reference (\( R \)), the assumption (\( A \)), and the probability (\( p \)).

- **Record a Security Claim.** Clicking on this tab presents an entry screen that prompts the user for a security specification (two fields: a functional requirement and an operational requirement 4.2.1), a field for an assumption, and a field for a probability. There is no need for a property field, since the property is predetermined by the choice of tabs.

- **Record Cost Information.** As we recall, there are two kinds of cost information that we want to record: failure cost, and verification cost. Depending on the user’s selection, the system presents a spreadsheet with four columns (Property, Reference, Cost,
Unit — for failure cost), or six columns (Property, Reference, Method, Assumption, Cost, Unit — for verification cost). This information is stored in tabular form to subsequently answer queries on failure costs or verification costs.

- **Record Domain Knowledge.** Because dependability claims are formulated using domain-specific notations, a body of domain-specific knowledge is required to highlight relevant properties and relationships, and to enable the inference mechanism to process queries. This domain knowledge is recorded by selecting the appropriate tab on the system.

- **Queries.** Clicking on the tab titled Submit Query prompts the user to select from a list of query format. The only format that is currently implemented is titled Validity of a Claim, and its purpose is to check the validity of a claim formulated as

\[ \Pi(P \sqsupseteq R|A) \geq p, \]

for some property \(\sqsupseteq\), reference (Specification) \(R\), Assumption \(A\), and probability \(p\). Notice that we do not have equality, but inequality; this feature can be used if we have taken a number of dependability measures and wish to check whether they are sufficient to allow us to claim that \(P\) refines \(R\) with a greater certainty than a threshold probability \(p\).

To answer a query, the system composes a theorem that has the query as goal clause, and uses recorded dependability claims and domain knowledge as hypotheses. The theorem
prover we have selected for this purpose is *Otter* [21, 11, 12].

5.3 A Sample Demo

To illustrate the operation of the tool, we take a simple example. We will present, in turn, the dependability claims that we submit to this system, then the domain knowledge, and finally the query; this example is totally contrived and intends only to illustrate what we mean by composing diverse dependability claims. Also, even though the model that we envision has inference capabilities that are based on many types of rules (probabilistic identities, refinement rules, lattice identities, relations between various refinement properties, etc), in this demo we only deploy probabilistic rules.

For the purposes of this example, we summarily introduce the following notations, pertaining to a fictitious flight control system:

- **Specifications.** We consider a specification, which we call *nostall*, which represents the flight envelope of the aircraft that precludes stalling\(^2\). We also (naively) assume that this requirement can be decomposed into two sub-requirements, whose specifications, *Safepitch* and * Safepower*, represent requirements for safe pitch angles and safe power values.

- **Assumptions.** We assume (artificially) that the claims we make about refining specifications *Safepitch* and * Safepower* are contingent upon a combination of conditions

\(^2\)stalling is the condition that arises when a combination of low power and/or low speed and/or high pitch cause the aircraft to to have less lift than weight and to nose-dive and fall.
that involve two predicates: \textit{elevator}, which represents the property that some module that interprets elevator control signals is correct; and \textit{throttle}, which represents the property that some module that interprets throttle control signals is correct.

Using these notations, we illustrate the deployment of the tool by briefly presenting the dependability claims, the domain knowledge, then the query that we submit to it.

- \textit{Claims}. Using the system’s GUI screens, we enter the following claims, where $P$ represents the flight control system:

\[
\Pi(P \models \text{Safepitch}|\text{elevator}) = 0.98.
\]

\[
\Pi(P \models \text{Safepitch}|(\neg \text{elevator} \land \text{throttle})) = 0.95.
\]

\[
\Pi(P \models \text{Safepitch}|(\neg \text{elevator} \land \neg \text{throttle})) = 0.93.
\]

\[
\Pi(P \models \text{Safepower}|\text{elevator}) = 0.95.
\]

\[
\Pi(P \models \text{Safepower}|\neg \text{elevator}) = 0.90.
\]

- \textit{Domain Knowledge}. We submit the following domain knowledge under the form of predicates, where $\text{indep}(p,q)$ means that events $p$ and $q$ are independent; one could questions whether some of the claims of independence are well-founded, but we make these assumptions for the sake of simplicity.

\[
\text{indep}(\text{elevator}, \text{throttle}).
\]
\[ \text{indep}(P \sqsupset \text{Safepitch}, P \sqsupset \text{Safepower}). \]

\[ P \sqsupset \text{Nostall} \iff (P \sqsupset \text{Safepitch} \land P \sqsupset \text{Safepower}). \]

- **Query.** We submit the query whether the following claim

\[ \Pi(P \sqsupset \text{Nostall}|A) \geq 0.90, \]

is valid, where \( A \) is the assumption that the probability of \textit{elevator} is 0.90 and the probability of \textit{throttle} is 0.80.

The system generates a theorem and submits it to Otter; then it analyzes the output file to determine if a proof was produced. The claim is deemed to be valid.

In the long run, we may choose a different theorem prover, or a different means to infer queries from claims than theorem provers altogether. Whereas theorem provers are adequate for symbolic manipulations, what we need in our type of application is a combination of symbolic manipulation and numeric calculations. We have resolved this matter in this simple case by running two parallel inference threads, in a way, by declaring arithmetic operations to be evaluable (rather than simply symbolic), and adding clauses such as

\[
\begin{align*}
\text{all } x \ y \ z \ ((x+y)=z) & \iff \text{sum}(x,y,z)). \\
\text{all } x \ y \ z \ ((y=(z-x)) & \iff \text{sum}(x,y,z)). \\
\text{all } x \ y \ z \ (\text{sum}(y,x,z) & \iff \text{sum}(x,y,z)).
\end{align*}
\]

to support symbolic equation manipulations and simplifications.
6 Conclusion

In this paper we have considered past work that attempts to compose eclectic verification claims and decompose aggregate verification goals, and have attempted to extend it. We have attempted to extend it by encompassing more dimensions of dependability, acknowledging the probabilistic nature of claims and goals, integrating failure and verification costs, and highlighting relationships between diverse dimensions of dependability. The result is a set of research challenges and research avenues that we have motivated and illustrated, but hardly explored.

References


