Quantum Annealing as an alternative heuristic for solving combinatorial optimization problems

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## Combinatorial optimization

- Optimize a cost function over a finite (but usually very large) set
- In many cases, best known algorithms scale exponentially with problem size
- Many important practical problems are of this form

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<th>Problem</th>
<th>Application</th>
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<td>Traveling salesman</td>
<td>Logistics, vehicle routing</td>
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<td>Minimum Steiner tree</td>
<td>Circuit layout, network design</td>
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<td>Graph coloring</td>
<td>Scheduling, register allocation</td>
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<td>MAX-CLIQUE</td>
<td>Social networks, bioinformatics</td>
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<td>QUBO</td>
<td>Machine learning, software V&amp;V</td>
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<td>Integer Linear Programming</td>
<td>Natural language processing</td>
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<td>Sub-graph isomorphism</td>
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<td>Job shop scheduling</td>
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<td>Motion planning</td>
<td>Robotics</td>
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<td>MAX-2SAT</td>
<td>Artificial intelligence</td>
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Hardness of CO

- Local minima “trap” many algorithms
- Discrete nature of state space
Quantum Annealing (QA)

Quantum Tunneling

$E$

$\Delta E$

$x$

$e^{-x\sqrt{\Delta E}}$

$x^2 \Delta E \begin{cases} \ll 1 & \text{easy to tunnel} \\ \gg 1 & \text{hard to tunnel} \end{cases}$
Quantum Annealing (QA)

Use quantum fluctuations (by tuning a non-commuting field) to escape local minima

**Quantum Hamiltonian**

\[ H(t) = -A(t) \sum_{i=1}^{N} \sigma_i^x + B(t) H_{\text{Ising}} \]

Start at \( t = 0 \)

\( B(0) = 0 \)

Initialize in ground state of \( H(0) \)

End at \( t = t_f \)

\( A(t_f) = 0 \)

End with only \( H(t_f) \)

\[ |\psi(0)\rangle = |\varepsilon_0(0)\rangle \] \( \approx \)

\[ |\psi(t_f)\rangle \approx |\varepsilon_0(t_f)\rangle \]
QA as a heuristic for CO

- QA requires less quality of resources than full universal QC
- We have a (candidate) QA implemented
- We need to better understand the usefulness and power of this intermediate step to QC

Some possible uses for QA

- Fast approximates solutions
- Sampling space of solutions (e.g., ALLSAT)
- Sampling distributions
- Complement classical algorithms
QA as a heuristic for CO

- **Fast approximate solutions**: QA may require long times to get exact solution due to errors, but good quality solutions can be generated extremely fast (e.g., usec).

- **Sampling of solution space**: some problems require finding as many solutions as possible (e.g., V&V). QA could be designed to sample different regions of the solution space than other classical methods.
QA as a heuristic for CO

• **Sampling distributions**: adjust parameters of QA to approximately sample from different distributions

  \[ \text{QA} (\lambda_i) \rightarrow P(x ; \{\lambda_i\}) \]

• **Complement classical algorithms**: combine sampling capabilities and fast generation approximate solutions as subroutines of classical approaches
Summary

• Quantum annealers aim at exploiting quantum mechanics to avoid certain pitfalls of classical optimization methods

• Quantum annealers will likely be easier to develop than a full QC

• Even though they have restricted capabilities, they can still provide a useful complement to classical approaches to combinatorial optimization

• Studying these advantages can also guide us in how to design quantum annealers tailored for different applications (like approximate sampling)