How Powerful are Adiabatic Optimization Algorithms?

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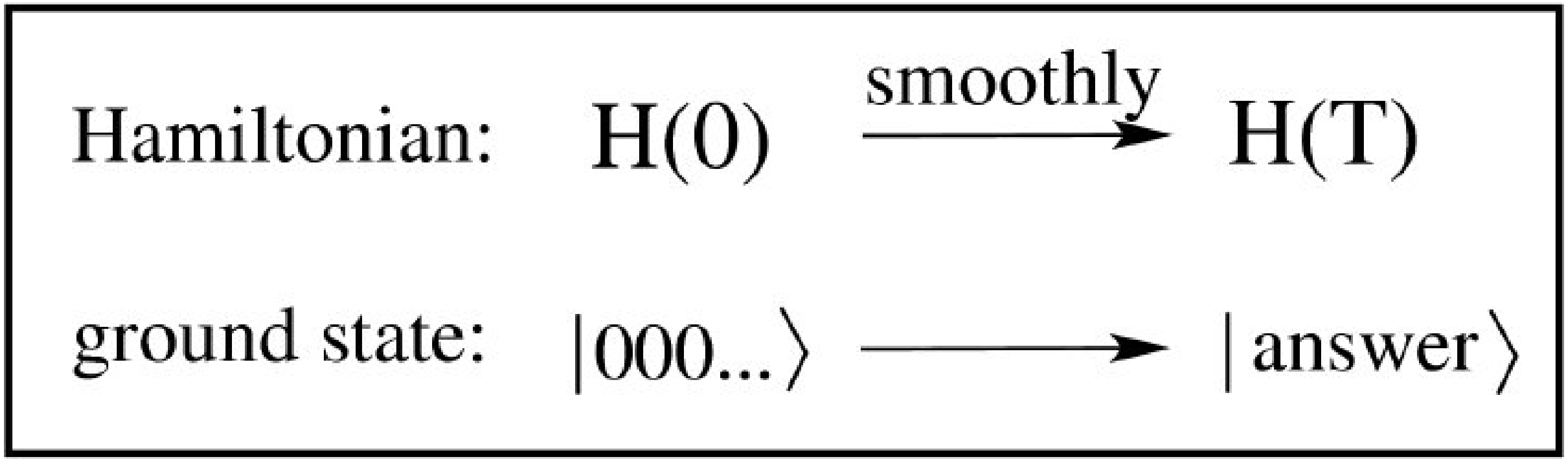
Adiabatic Quantum Computing



- Proposed in 2000:

  [quant-ph/0001106. Farhi, Gutman, Goldstone, Sipser]

- By adiabatic theorem [Elgart, Hagedorn, 2012]:

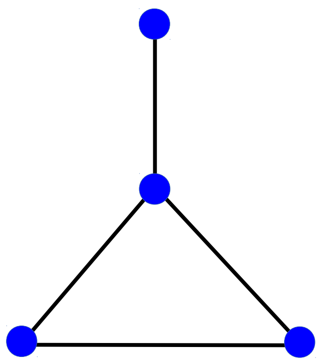
$$T = \widetilde{O}\left(\frac{1}{\gamma^2}\right)$$

$\gamma =$ eigenvalue gap above ground energy

- Naturally suited to optimization problems

Adiabatic Optimization

- Let G be a graph. Let L_G be its Laplacian.



$$L_g = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 2 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$

- Let W be a potential on the vertices.

$$W = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

- Adiabatic Algorithm to find minimum:

$$H(s) = (1-s)L_G + sW$$

Adiabatic Optimization

- Let G be a graph. Let L_G be its Laplacian

These Hamiltonians are *stoquastic*: all off-diagonal matrix elements are nonpositive.

Physics intuition: "no sign problem".

Theorem: Stoquastic AQC is in postBPP.
[Bravyi, DiVincenzo, Oliveira, Terhal, 2006]

- Le

$$W = \begin{bmatrix} \cdots \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

- Adiabatic Algorithm to find minimum:

$$H(s) = (1-s)L_G + sW$$

Can adiabatic optimization outperform classical optimization algorithms?

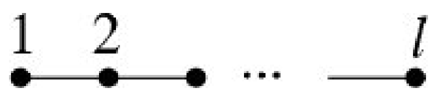
- For some problems adiabatic optimization beats simulated annealing.
  [Farhi, Goldstone, Gutmann, 2002] & [Reichardt, 2004]
- Provable exponential speedup for a query problem by similar algorithms that are not adiabatic.
  [Nagaj, Somma, Kieferova, 2012]
- For some problems adiabatic optimization is exponentially beaten by classical algorithms (even gradient descent).
  [van Dam, Mosca, Vazirani, 2001] & [Jarret, Jordan, 2014]
- Can achieve quadratic, Grover-like speedups:
  [Roland, Cerf, 2002] & [Somma, Boixo, 2013]

Analyzing the power of adiabatic optimization algorithms

	Strengths	Limitations
Numerical	• General • Exact	• Small Instances
Experimental	• Bigger than numerical • More general than analytic	• Hard to distinguish hardware limitations from algorithmic ones.
Analytic	• Asymptotic • Rigorous	• Hard, except for highly symmetric instances.

Tools From Spectral Theory

- Graph Laplacians are closely related to $-\nabla^2$

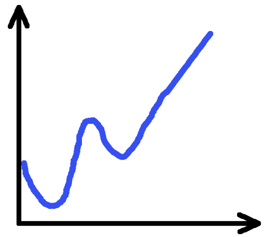


$$L_g = \begin{bmatrix} 1 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & & \ddots & \end{bmatrix}$$

- We want: $L_G + W$
- Traditional spectral theory gives us: $-\nabla^2 + W$
- Spectral graph theory gives us: L_G

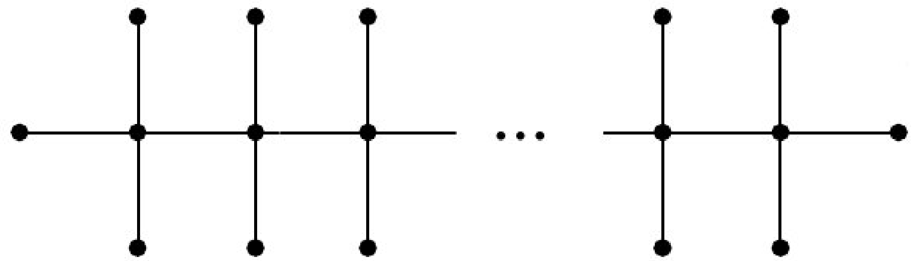
Local Minima

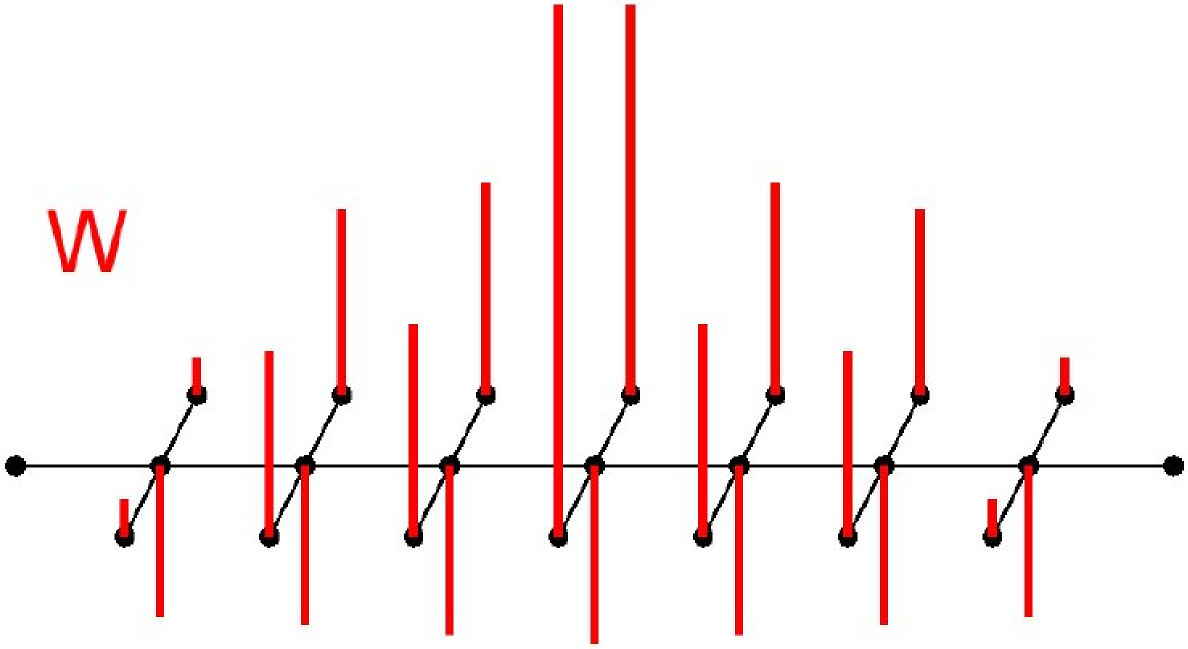
- Much discussion of adiabatic optimization focuses on tunneling out of local minima.
  - If barrier is broad tunneling is exponentially slow.
  - If barrier is tall, we may beat classical annealing.



- Underlying assumption: if there are no local minima, optimization runs fast.
- Is this true?

Counterexample





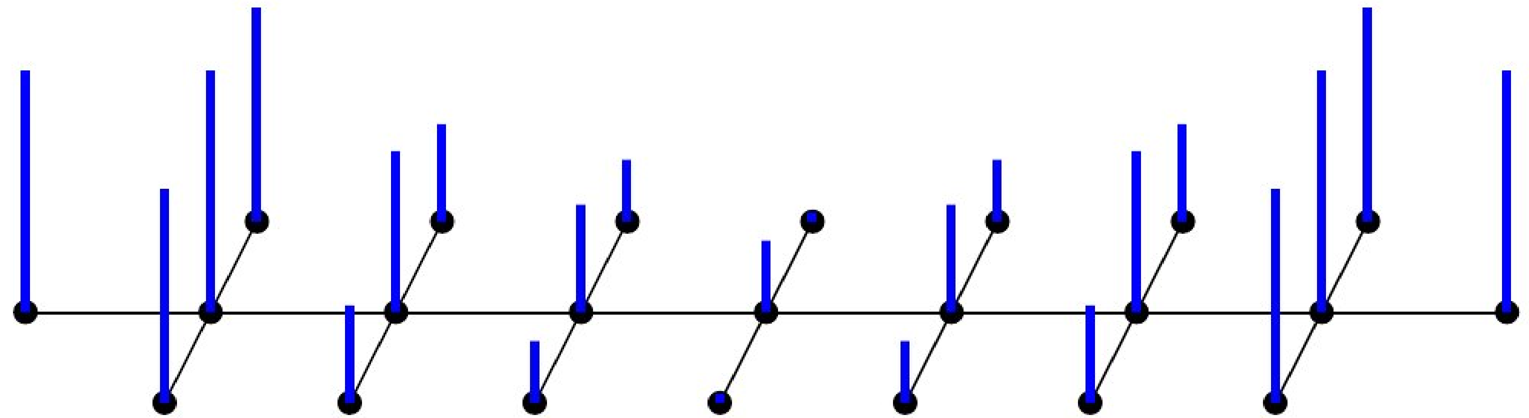
$$\gamma = 2^{-O(\ell)}$$

[Michael Jarret and S. Jordan, *Adiabatic optimization without local minima*. Quant. Inf. Comp. 14(3/4):181, 2014. arXiv:1405.7552]

Counterexample

Q. Why is the gap exponentially small?

A. Because the ground state is “lobed”.



Theorem: If the ground state is single-peaked the gap satisfies $\gamma = \Omega(1/|V_G|^2)$.

Tools

- The counterexample is by “bare hands.”
- The theorem for single-peaked ground states is by conductance.
- Tighter bounds on path and hypercube by Poincare's inequality.
- Optimal bounds on path and hypercube by variational methods.

  [Jarret, Jordan, J. Math. Phys. 55:052104, 2014]
- More general bounds from heat kernels.
  [To appear]

Tools

from spectral graph theory

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from spectral theory

Conclusion

- Two directions for adiabatic optimization research:
  - Applications
  - Complexity Theoretic Foundations
- Foundations
  - Local minima are important
  - They are not the whole story
  - Challenge: Construct an unambiguous example of superpolynomial speedup by adiabatic quantum optimization (even if contrived).