A Posteriori Error Analysis of Stochastic Differential Equations Using Polynomial Chaos Approximations

Mathematical and Computational Tools for Predictive Simulation of Complex Coupled Systems Under Uncertainty

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Abstract

Uncertainty quantification, broadly defined, is the study and assessment of uncertainties and errors that arise in the modeling and simulation of complex systems with the goal of improving confidence in simulation. Uncertainty and error are ubiquitous in predictive modeling and simulation due to unknown model parameters and various sources of numerical error. Consequently, there is considerable interest in developing efficient and accurate methods to quantify the uncertainty in the outputs of a computational model.

A simple, but inefficient, approach is Monte Carlo simulation where (pseudo-)random samples of input distributions are generated and the model is solved for each sample resulting in samples of the output distribution. Statistics or density estimates on outputs are then generated from these samples. This approach is easy to implement, but is computationally prohibitive for complex models where a large number of samples are required and each sample requires a solution of the model. In recent years, stochastic spectral methods have become an increasingly popular way to approximate response surfaces and to reduce the computational burden of Monte Carlo. The objective is to construct polynomial chaos (PC) expansions for input variables and the solution to the computational model. The desired statistics on the outputs can be cheaply estimated by sampling the PC expansion. However, each of these samples contains numerical error due to spatial and temporal discretizations as well as the truncation of the PC expansion.

We use adjoint-based *a posteriori* error analysis to quantify the effect of numerical error on the PC expansion of a quantity of interest [2]. The advantage of this method is that by computing a PC representation of the forward and adjoint solutions we cheaply produce error estimates for any choice of the random parameter without additional forward/adjoint solves. This error estimate can be used to provide computable error bounds, generate adaptive sampling procedures, or to define an *improved linear functional* which converges at a much faster rate than the linear functional computed from the PC expansion of the solution [1].

References

[1] T. Butler, P. Constantine, and T. Wildey, *A posteriori error analysis of parameterized linear systems using spectral methods*. Submitted to SIAM J. Matrix Anal. Appl., 2011.

[2] T. Butler, C. Dawson, and T. Wildey, *A posteriori error analysis of stochastic differential equations using polynomial chaos expansions*, SIAM J. Scientific Computing, 33 (2011), pp. 1267–1291.