Positivity-preserving High-Order Runge-Kutta Discontinuous Galerkin Schemes with Applications to Shallow Water Equations

High Order Discontinuous Galerkin Method and Applications

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Abstract

Recently a general framework has been developed to obtain high order accurate Runge-Kutta discontinuous Galerkin (DG) or finite volume (FV) schemes which satisfy strict maximum principle for scalar multi-dimensional nonlinear conservation laws, incompressible flows in two-dimensional vorticity-streamfunction formulation, and passive convection in incompressible fields, and are positivity-preserving for density and pressure for compressible Euler equations. This general framework (for arbitrary order of accuracy) is established to construct a limiter for the DG or FV method with first order Euler forward time discretization, which is easy to implement and can be proved mathematically to maintain high order accuracy. Strong stability preserving (SSP) high order time discretizations will keep the same property and make the scheme uniformly high order in space and time. One remarkable property of this approach is that it is straightforward to extend the method to two and higher dimensions on arbitrary triangulations.

We will first survey this methodology, highlighting its fundamental ideas and applicability. We will then move on to discuss a recent work on developing this methodology for shallow water equations with dry areas. The resulting discontinuous Galerkin or finite volume schemes are high order accurate, can maintain positivity of water height, are well-balanced for still water steady state, and can treat dry areas in a conservative and stable fashion. This is a joint work with Yulong Xing and Xiangxiong Zhang.

References

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