Rare-event Splitting for Efficient Simulation of Cascading Blackouts

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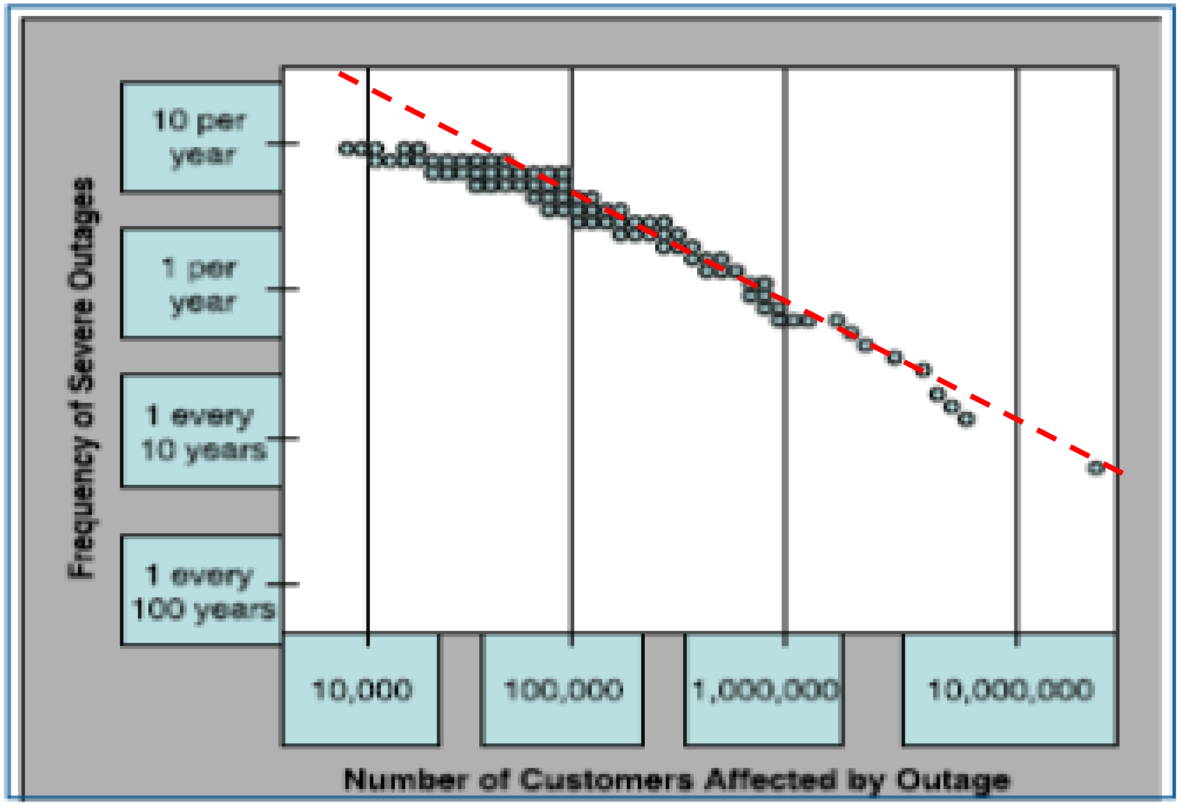
Rare Events in Power Grids

Individual Outages in North America, 1984-1997



Power-tailed Distribution

$$\Pr\{X > x\} \approx cx^{-a}$$

Source: U.S.-Canada Power System Outage Task Force. 2004. Final Report on the August 14, 2003 Blackout in the United States and Canada: Causes and Recommendations. Adapted from John Doyle. 1999. Complexity and Robustness.

Problems Simulating Rare Events

$$\gamma \equiv \text{Probability of rare event}$$

$$\hat{\gamma}_n \equiv \frac{\text{\# replications in which rare event occurs}}{n}$$

Relative Error $$RE[\hat{\gamma}_n] \equiv \frac{\sqrt{\text{var}[\hat{\gamma}_n]}}{E[\hat{\gamma}_n]} = \frac{\sqrt{\gamma(1-\gamma)/n}}{\gamma} \approx \frac{1}{\sqrt{n\gamma}}$$

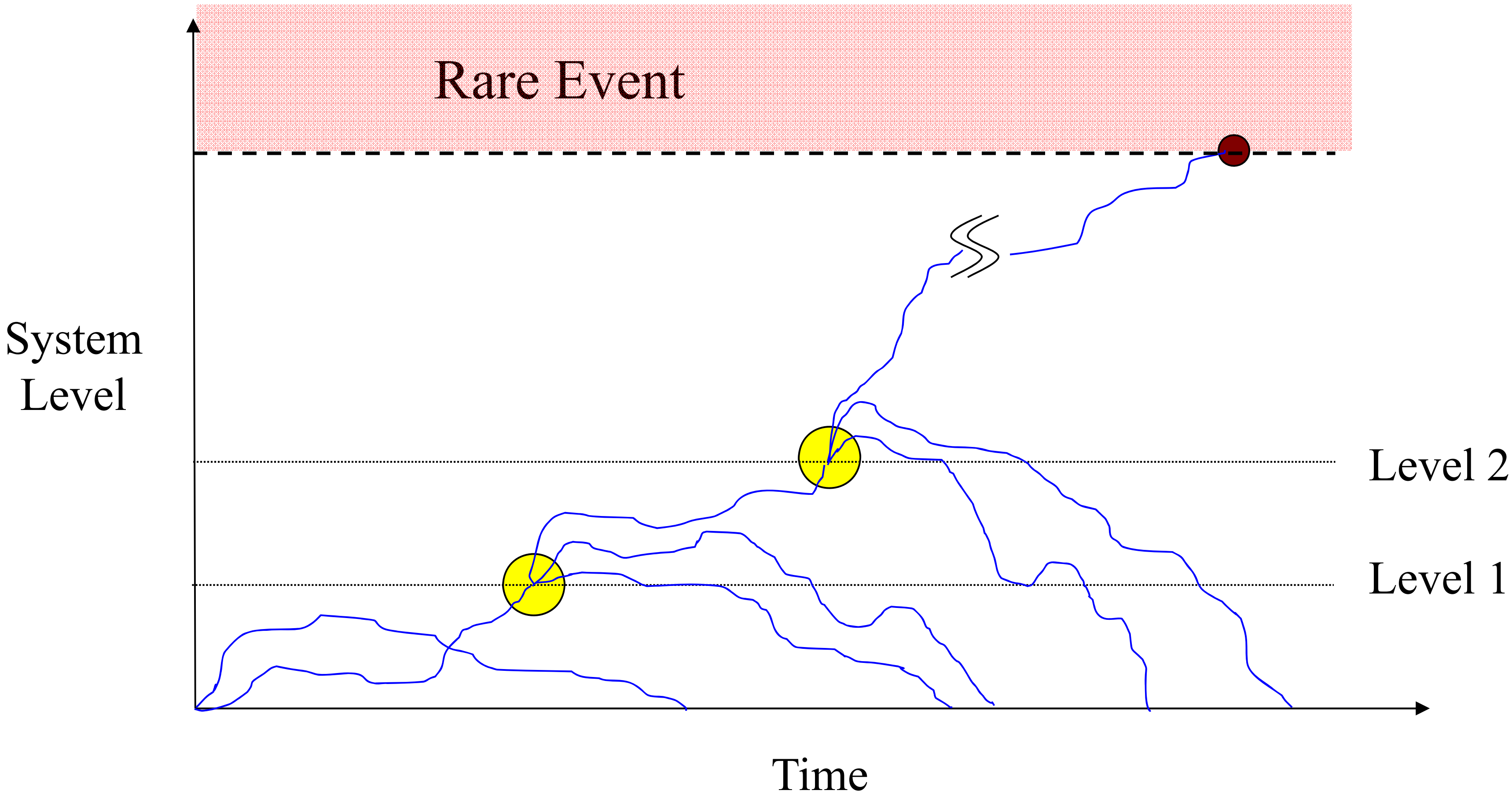
Computer time to simulate rare events can be prohibitive

Time Required to Achieve 1% Relative Error
(Assumes 1,000 replications per second)

Rare Event Probability γ	**Simulation Runs** n	**Required Time**
10^{-3}	10^{7}	16.7 minutes
10^{-5}	10^{9}	1.2 days
10^{-7}	10^{11}	116 days
10^{-9}	10^{13}	31.7 years

Splitting

Splitting is a technique to improve the efficiency of rare-event simulation



E.g.: Garvels, M. 2000 The splitting method in rare event simulation. Ph.D. thesis University of Twente, The Netherlands.

Optimal Splitting

Minimize variance subject to fixed computing budget

Problem

Minimize: $\text{var}[\hat{\gamma}]$

such that: $b_1 n_1 + \ldots + b_m n_m \leq T$

n_i = number of runs for stage i
b_i = average computing time for stage-i simulation
p_i = probability of advancing from level i-1 to level i

Solution

Assuming probability of advancing from level j-1 to level j does not depend on starting state from level j-1, the optimal allocation satisfies:

$$n_1 b_1 \left(1 + \frac{n_1 p_1}{1 - p_1}\right) = n_2 b_2 \left(1 + \frac{n_2 p_2}{1 - p_2}\right) = \cdots = n_m b_m \left(1 + \frac{n_m p_m}{1 - p_m}\right)$$

Shortle, J., C. Chen, B. Crain, A. Brodsky, D. Brod, 2011. Optimal splitting for rare-event simulation. To appear in *IIE Transactions*.

Splitting: Multiple Designs

Problem

Maximize: $\min_{n_{ij}} \Pr\{\hat{\gamma}_1 < \hat{\gamma}_2, \hat{\gamma}_1 < \hat{\gamma}_3, \ldots, \hat{\gamma}_1 < \hat{\gamma}_n\}$

such that: $\sum_{i=1}^{n}\sum_{j=1}^{m} b_{ij} n_{ij} = T$

Maximize prob. of choosing lowest rare-event probability

b_{ij} = average time to simulate design-*i* stage-*j*
n_{ij} = number of runs for design-*i* stage-*j*
p_{ij} = prob. of advancing from level *j*-1 to *j*, design *i*
T = computing budget
$\hat{\gamma}_i$ = estimator for rare-event probability, design *i*

Solution (2 designs): Optimal allocation satisfies:

$$\frac{n_{ij}^2 b_{ij} p_{ij}}{\gamma_i^2 (1-p_{ij}) \prod_{r \neq j}\left(1 + \frac{1-p_{ir}}{n_{ir} p_{ir}}\right)} = \frac{n_{kl}^2 b_{kl} p_{kl}}{\gamma_k^2 (1-p_{kl}) \prod_{r \neq l}\left(1 + \frac{1-p_{kr}}{n_{kr} p_{kr}}\right)}$$

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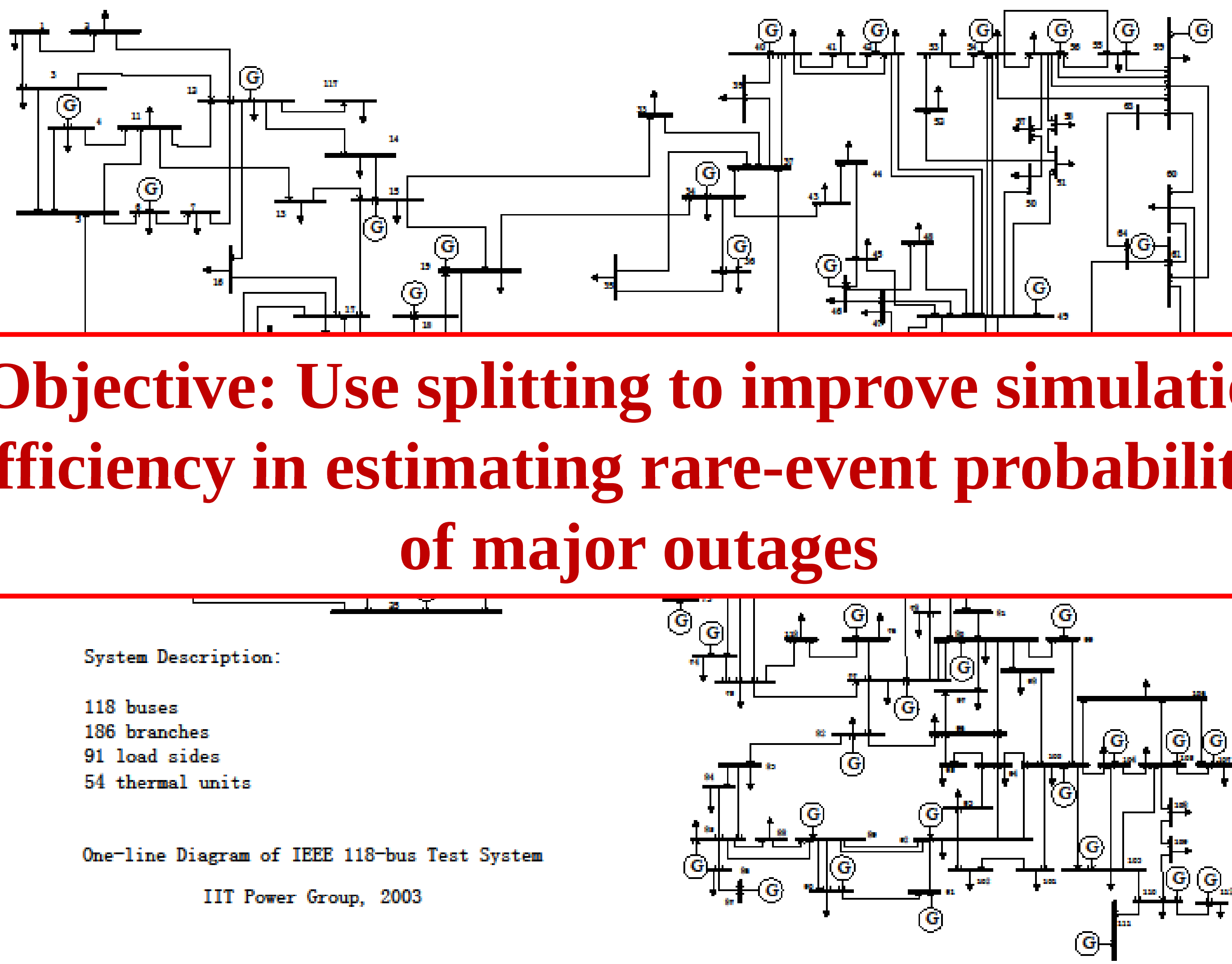
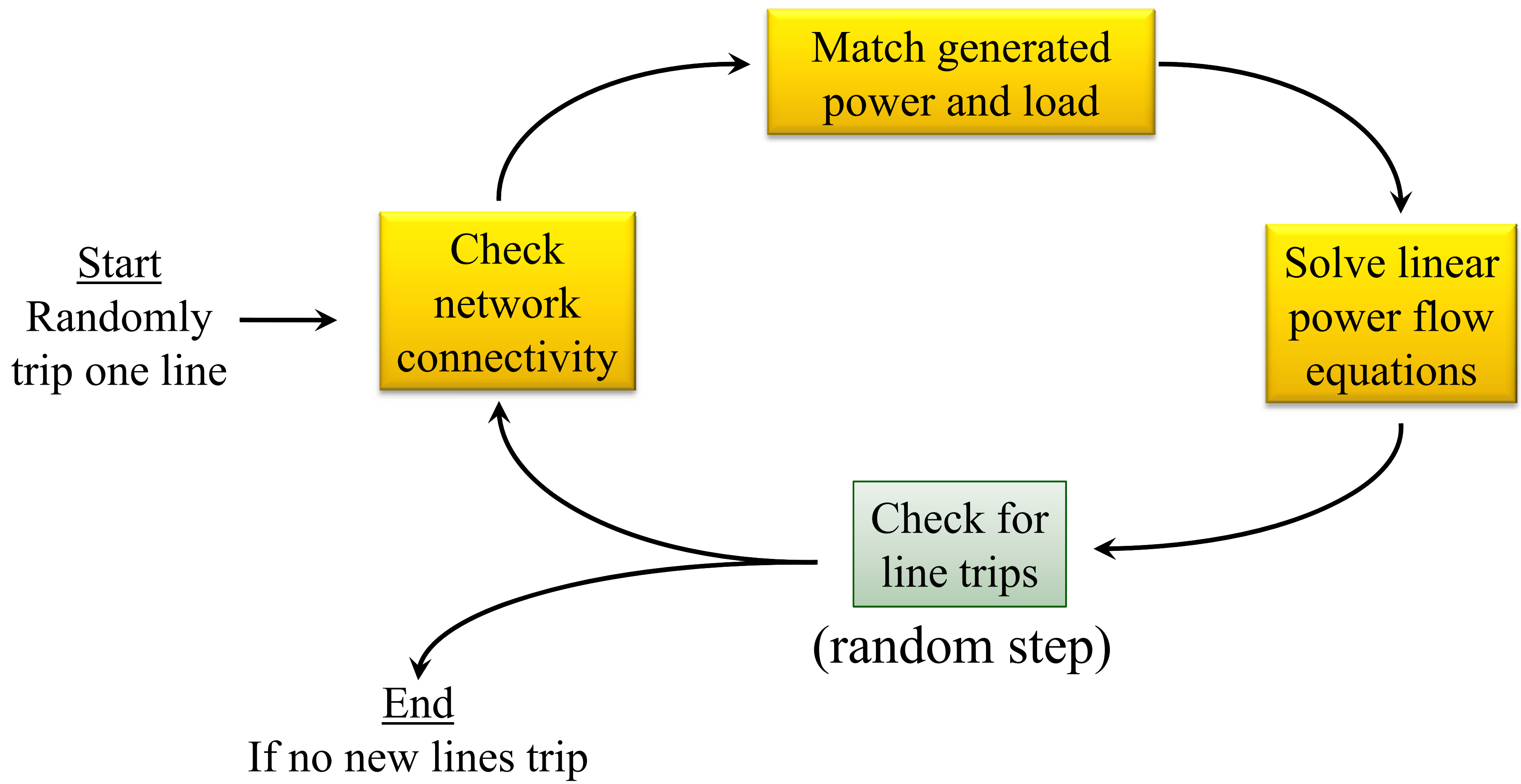
Application to Power Grids



Objective: Use splitting to improve simulation efficiency in estimating rare-event probabilities of major outages

IEEE 118-bus Test System

Blackout Model



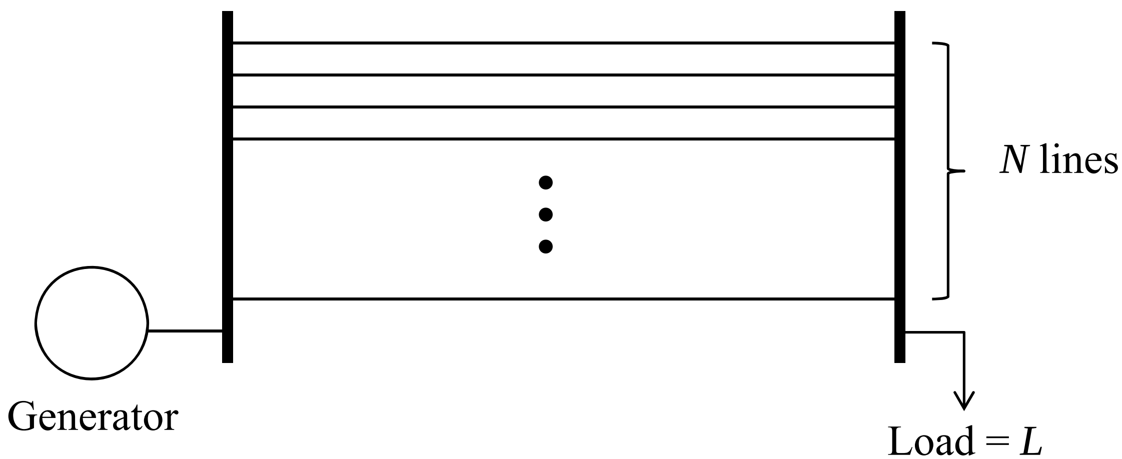
Similar to some models in literature, for example:

• Chen, J., J. Thorp, I. Dobson. 2005. Cascading dynamics and mitigation assessment in power system disturbances via a hidden failure model. *Electrical Power & Energy Systems*, 27, 318-326.

• Bae, K. J. Thorp. 1999. A stochastic study of hidden failures in power system protection. *Decision Support Systems*, 24, 259-268.

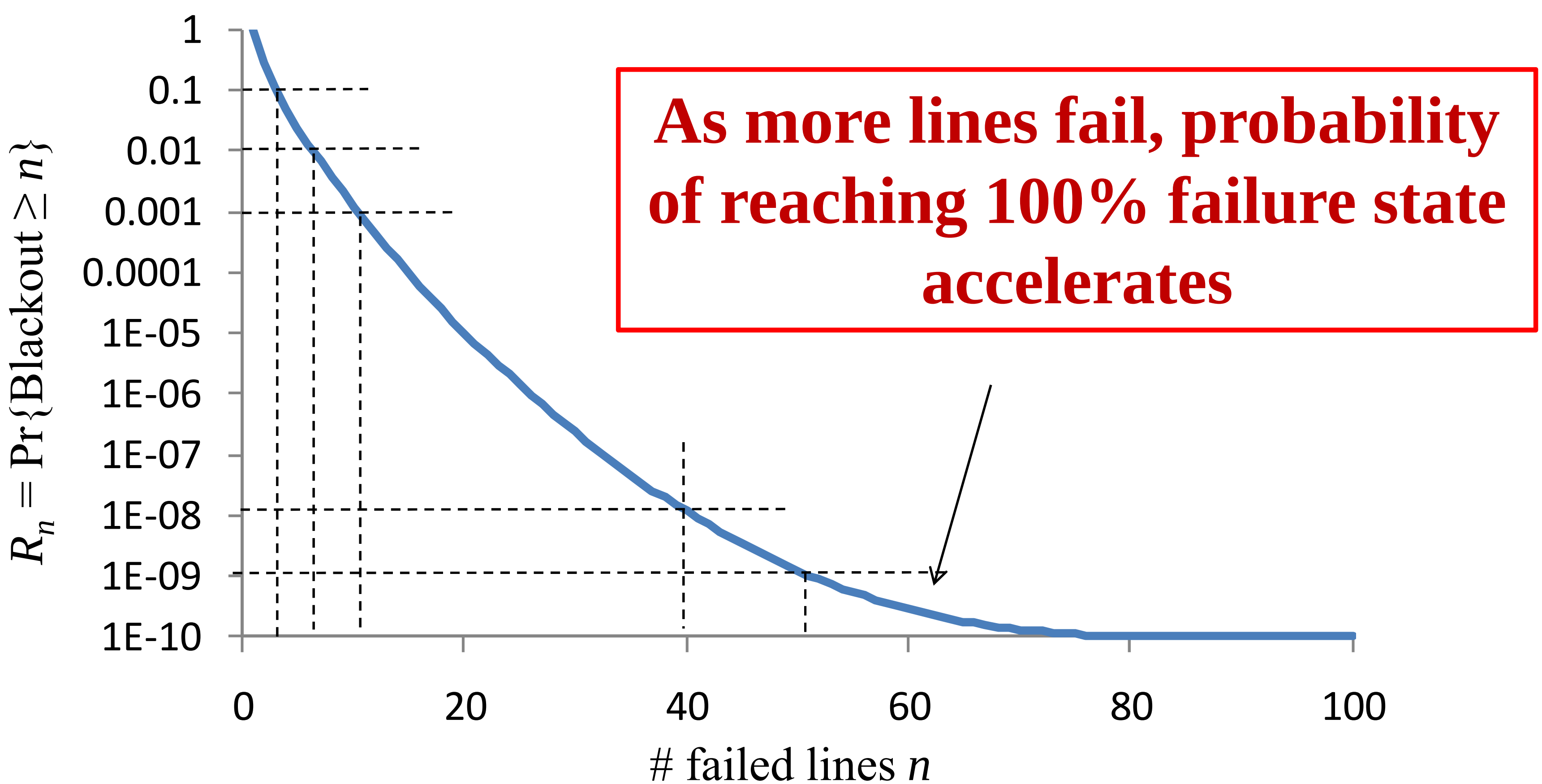
Example: Simple Model

- *N* identical parallel lines connecting two buses
- When a line fails, its load is equally distributed among the remaining lines
- Can be solved analytically as a Markov chain.
- Results obtained for simple network provide insight into application of splitting method for more complex networks



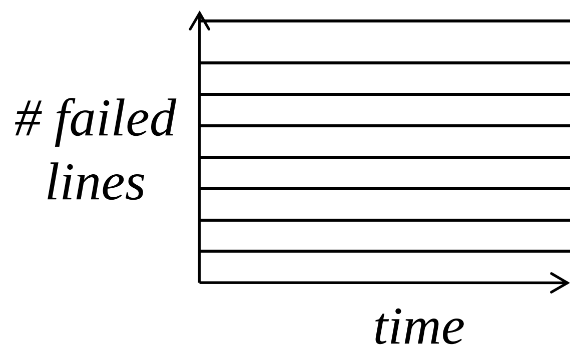
Similar to analytical model in Dobson, Carreras, Newman (2005)

Blackout-size Distribution



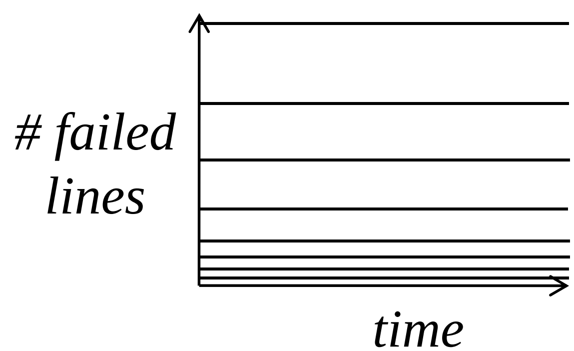
Splitting: Choice of Levels

- Evenly spaced by distance



Levels are evenly spaced

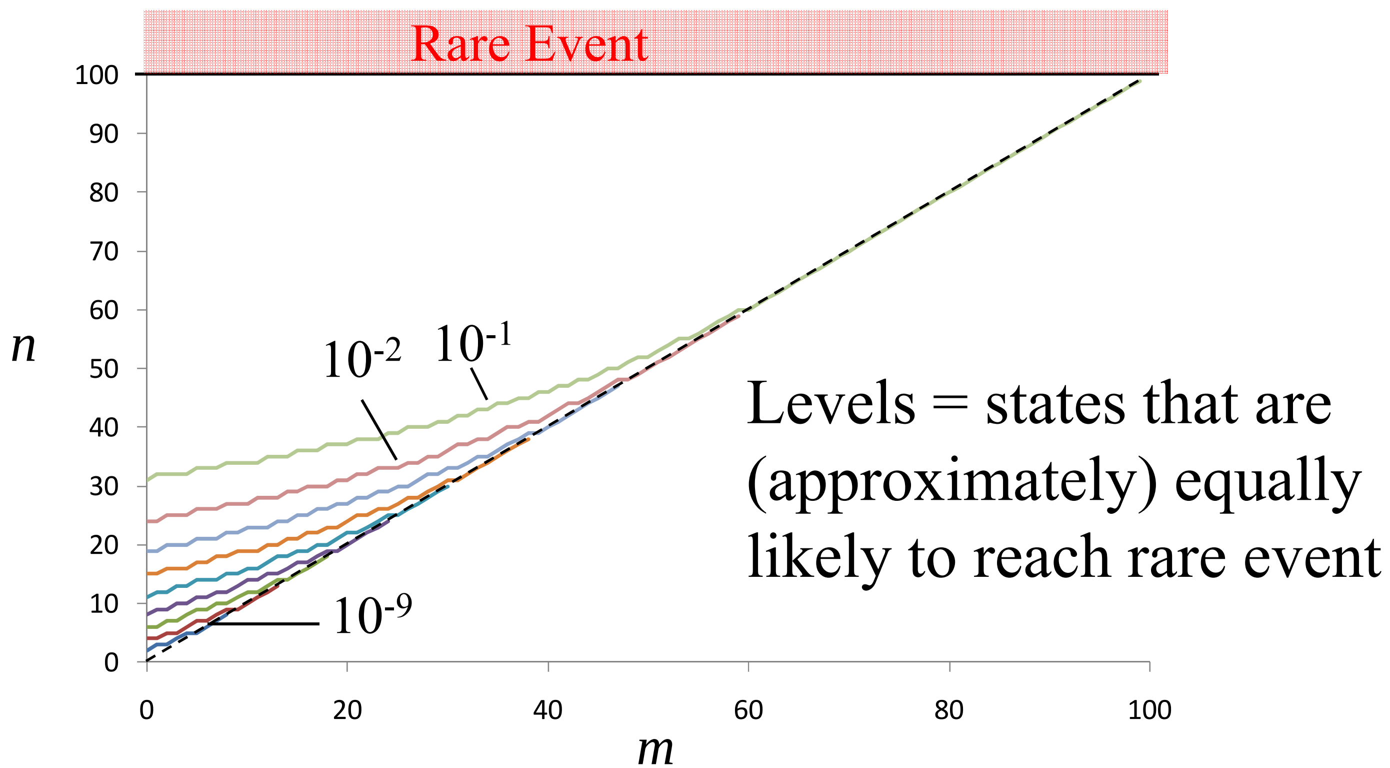
- Evenly spaced by probability



Probability of advancing from one level to the next is approximately the same (cascading effect implies greater spacing at higher levels)

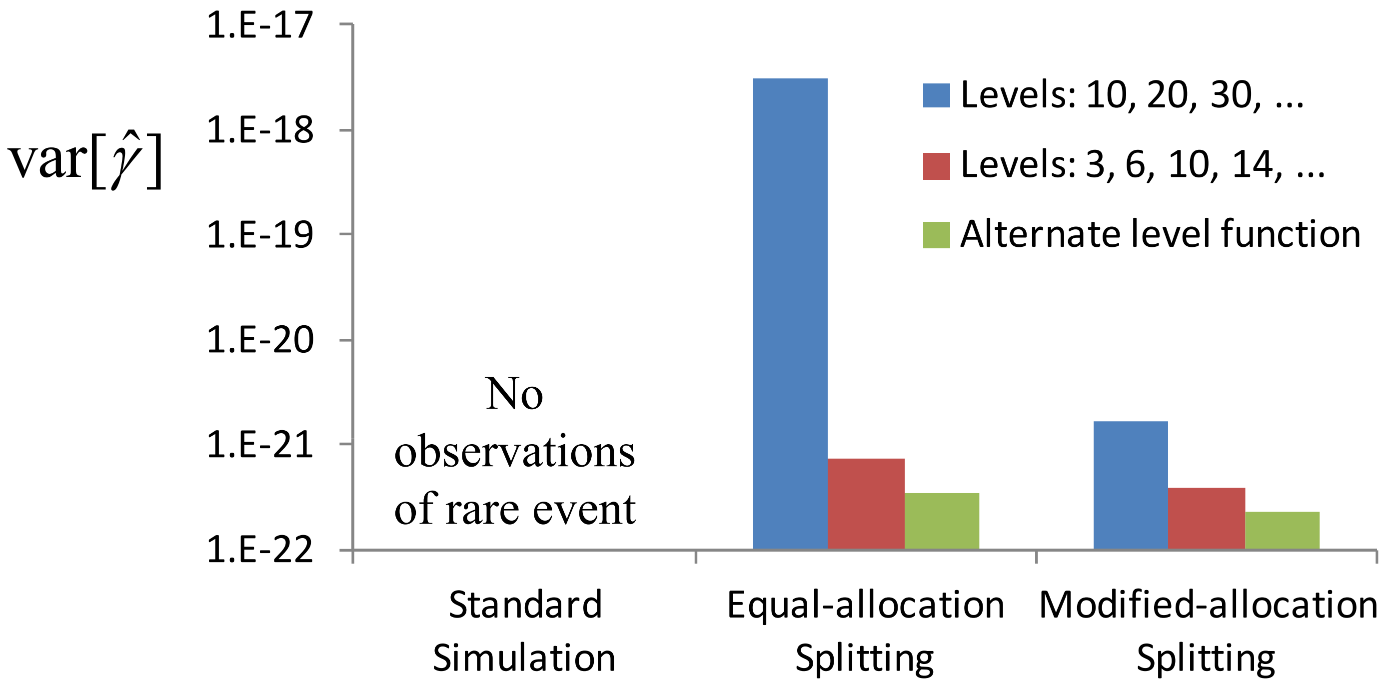
Alternate Level Function

- System state: (m, n)
  - n = # of presently failed lines
  - m = # of failed lines in previous iteration



Simulation Efficiency

Objective: Estimate $\gamma = \Pr\{\text{all lines fail}\}$



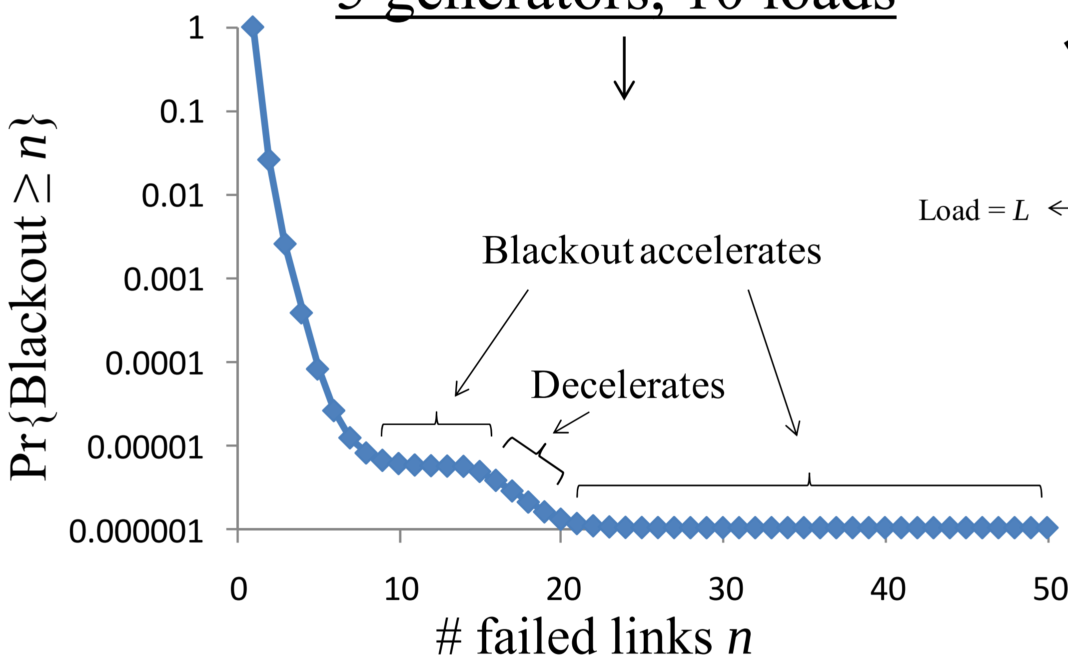
- **Standard simulation inadequate**
- **Levels evenly spaced by prob. better than evenly spaced**
- **Modified allocation better than equal-allocation splitting**

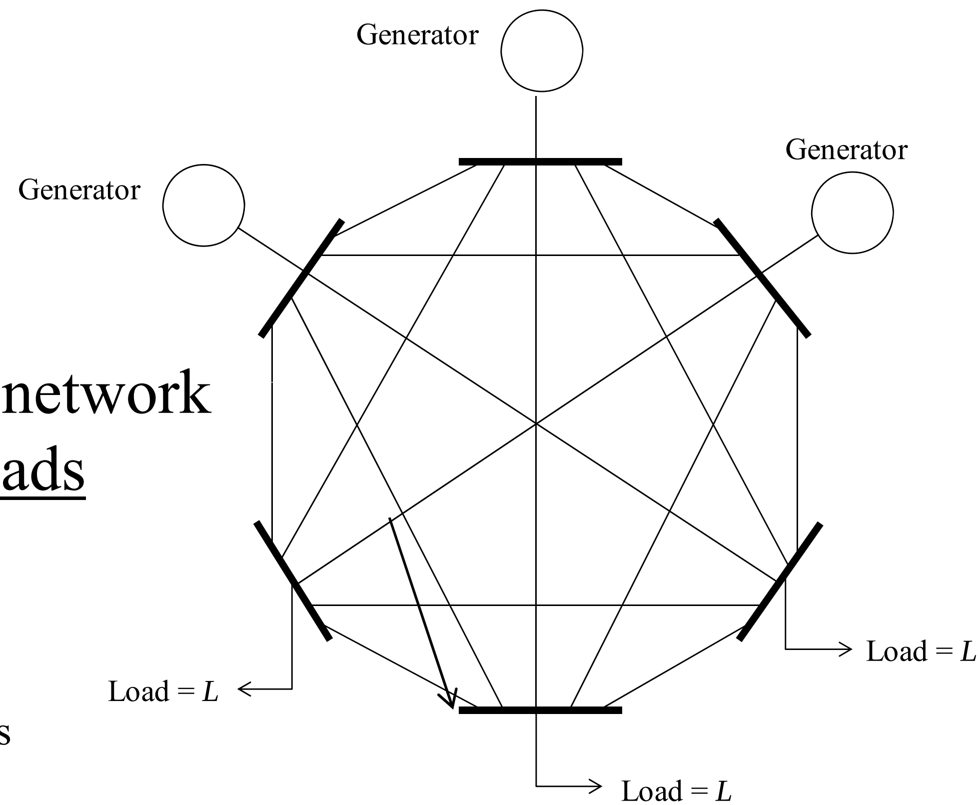
Prob. of advancing from level *j*-1 to *j* depends on starting state from level *j*-1 (modified allocation not necessarily optimal)

Example: Mesh Network

Example: 15-bus mesh network

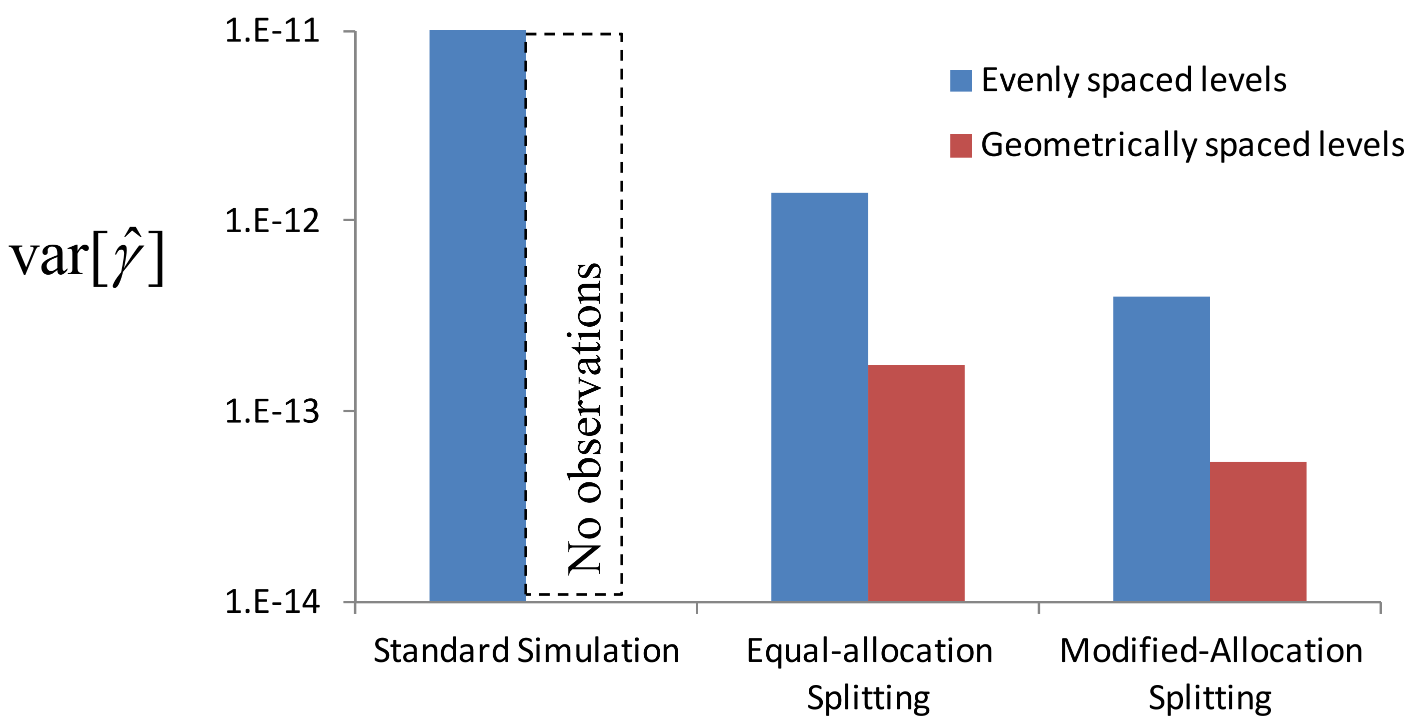
<u>5 generators, 10 loads</u>





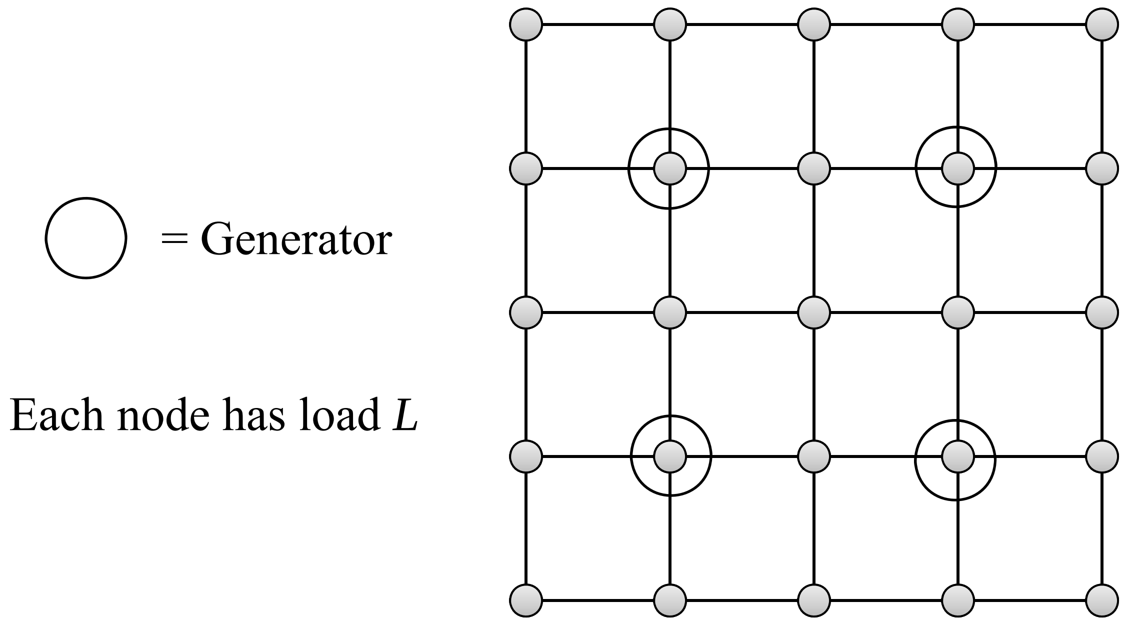
Simulation Efficiency

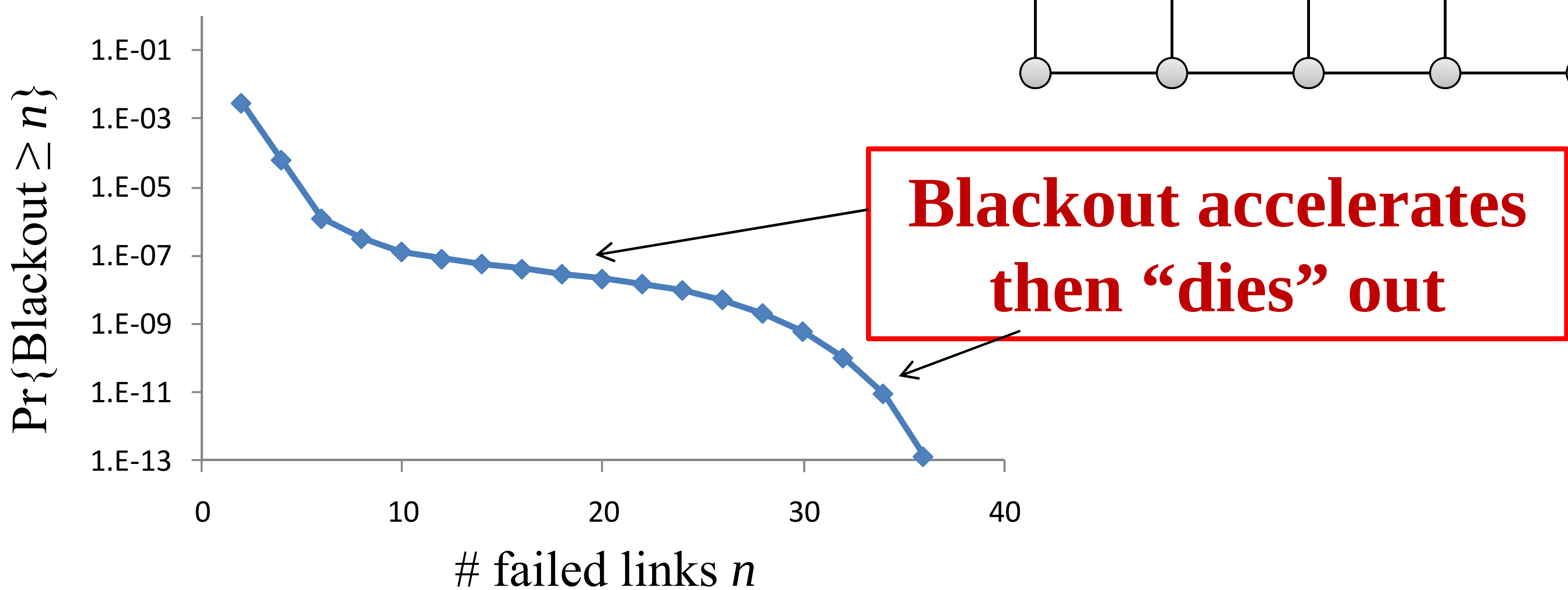
Objective: Estimate $\gamma = \Pr\{50 \text{ lines fail}\}$



Prob. of advancing from level *j*-1 to *j* depends on starting state from level *j*-1 (modified allocation not necessarily optimal)

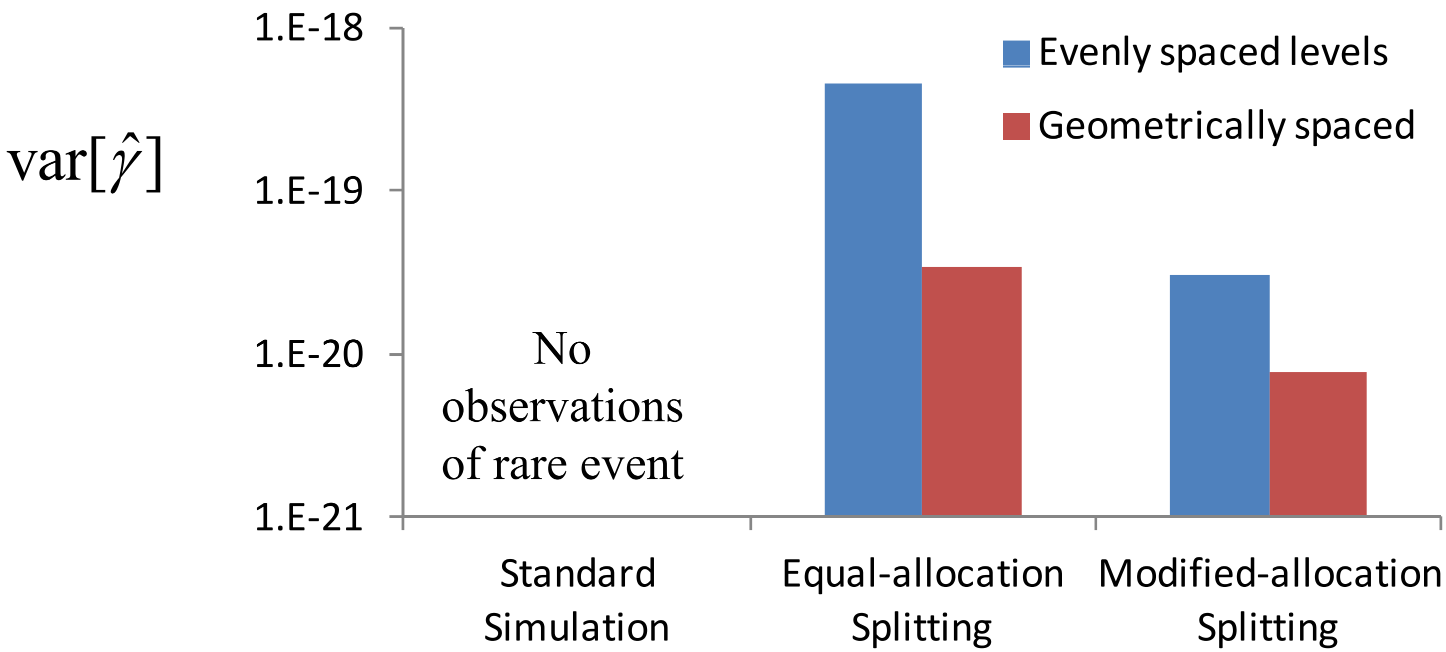
Example: Grid Network





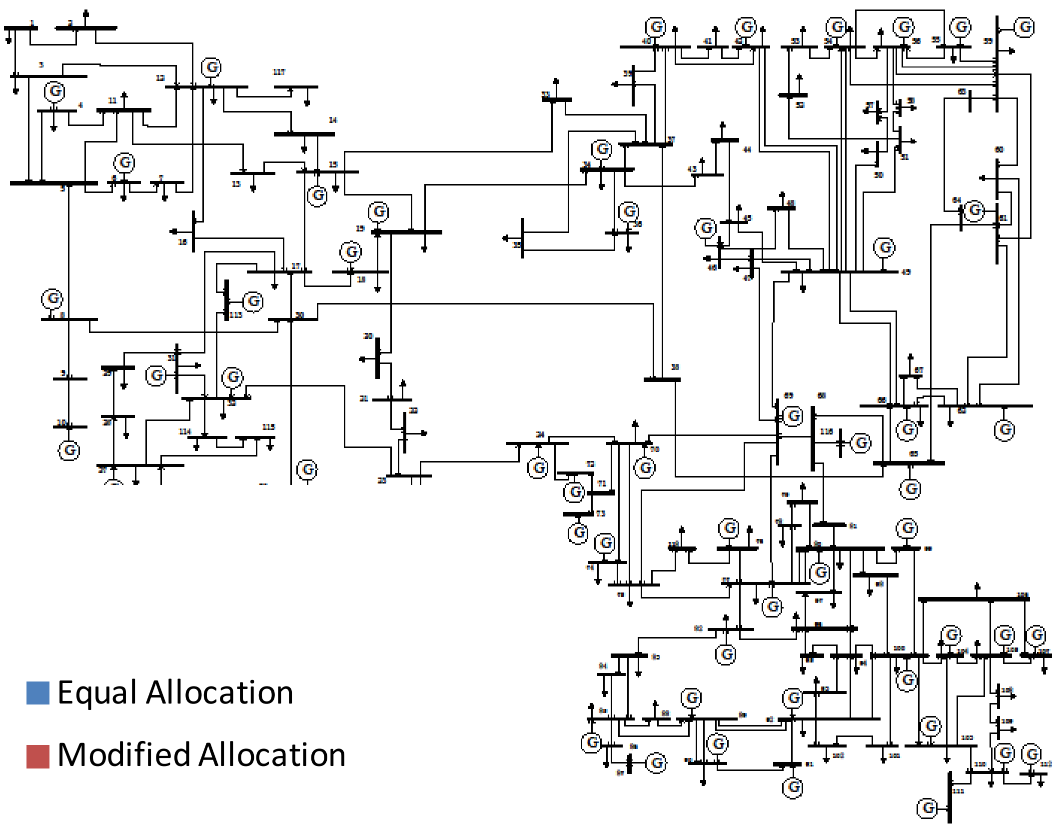
Simulation Efficiency

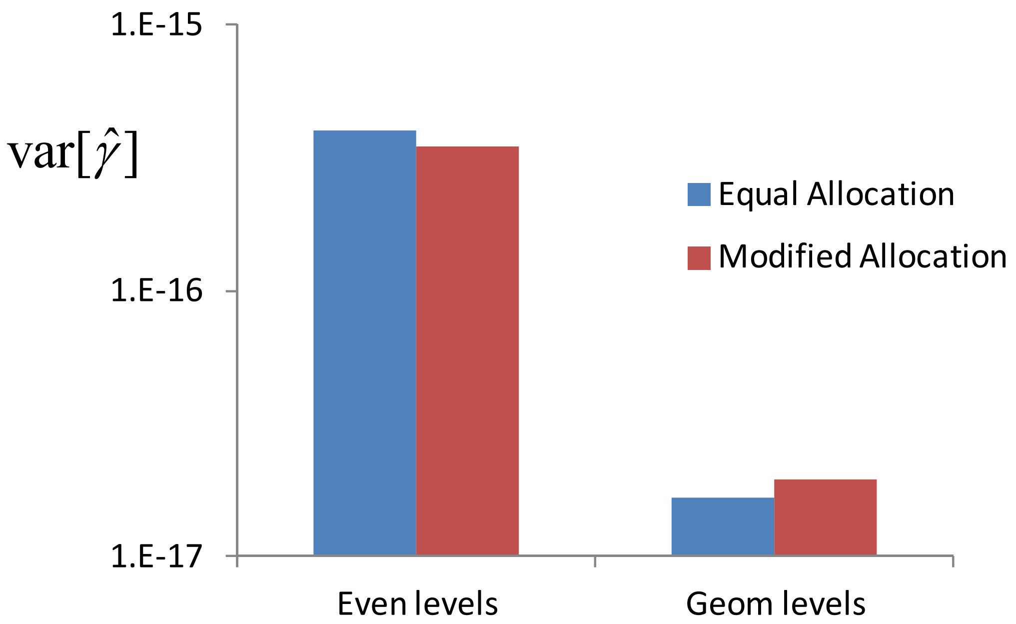
Objective: Estimate $\gamma = \Pr\{32 \text{ lines fail (out of 40)}\}$



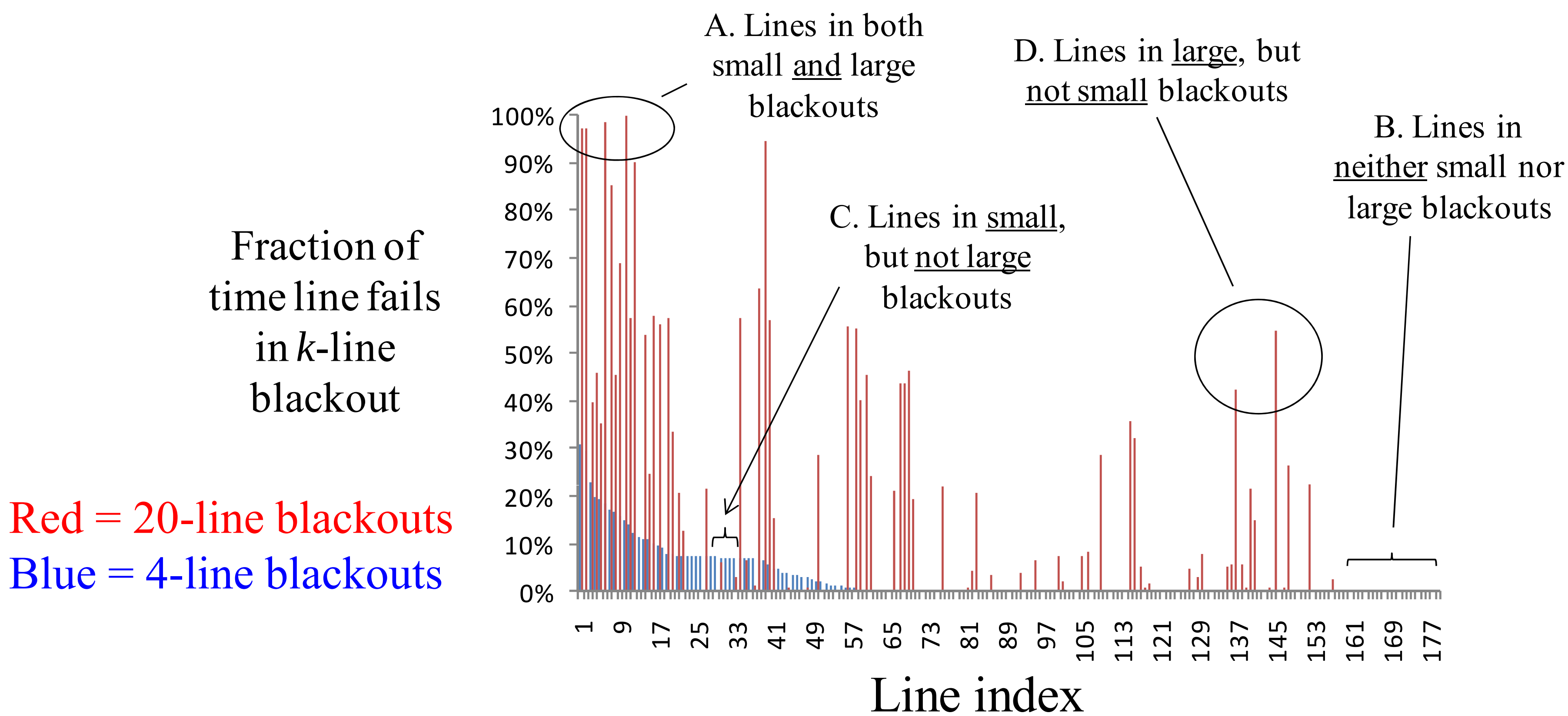
Prob. of advancing from level *j*-1 to *j* depends on starting state from level *j*-1 (modified allocation not necessarily optimal)

Example: 118-bus System





Line Failures in 20-Line Blackout



Alternate level functions: Weight line failures by:

- Power flow through line
- Number lines connected to failed line
- Fraction of time line fails in *k*-line blackout

Summary and Conclusions

- Allocation method for computing budget in rare-event splitting
- Application to model of stochastic cascading line failures
  - Simple analytical network. Choice of levels more important than choice of level function. Modified allocation method provides variance reduction.
  - Alternate models: Cascading nature of blackouts suggests levels with increasing spacing. Modified allocation method generally provides variance reduction