ReALE - Reconnection-Based ALE

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Arbitrary Lagrangian-Eulerian (ALE) Methods

- Explicit Lagrangian (solving Lagrangian equations) phase — grid is moving with fluid

- Rezone phase — changing the mesh (improving geometrical quality, smoothing, adaptation) — mesh movement

- Remap phase (conservative interpolation) — remapping flow parameters from Lagrangian grid to rezoned mesh
Rayleigh-Taylor Instability — Limitation of ALE

Solution: ReALE - Reconnection-based ALE

Rezone phase — Mesh is allowed to change connectivity
ReALE — Reconnection-based ALE

- **Lagrangian phase** — General polygonal meshes
- **Rezone phase** — Allows mesh reconnection
- **Remap phase** — Remapping from one polygonal mesh to another

*The Devil is in the Details*
Multimaterial Lagrangian Hydrodynamics

● Method of Concentrations
  * Transport equations for evolving concentrations, rescaling.
  * Mixture assumption - iso-pressure, iso-temperature, leads to effective $\gamma$ for mixture of ideal gasses

● Volume-of-fluid (VOF); Moment-of-Fluid (MOF)
  ○ Interface Reconstruction
  ○ Closure models for mixed cells
**ReALE - Flowchart**

**Initialization**
- Construction of initial Voronoi mesh $\text{Mesh } I^{n=0}$
- Cleaning
- Initial distribution of physical parameters $U^{n=0}$

$n := n+1$

**Lagrangian phase**
- Solving Lagrangian equations on general polygonal mesh
- Start $\text{Mesh } I^n$ $U^n$
- End $\text{Mesh } L^{n+1}$ $U^{n+1}$
- Same connectivity meshes

**Rezone phase**
- Definition of generator position for cell $c$:
  \[ \{ G^{n+1}_c \} \]
- Construction of associated Voronoi tessellation
- Cleaning
- $\text{Mesh } R^{n+1}$
- Resized mesh is a general polygonal mesh

**Remap phase**
- Remap from Lagrangian mesh onto Rezoned mesh
- $L^{n+1}$ $R^{n+1}$
- $U_L^{n+1}$ $U^{n+1}$
- Requires intersection-based remap

$n := n+1$

Start a new time step
Voronoi Tessellation - Definition

Set of generators: $g_i = (x_i, y_i)$
Voronoi cell: $V_i = \{ r = (x, y) : |r - g_i| < |r - g_j|, \text{ for all } j \neq i \}$

Centroid of the cell

$$c_i = \frac{\int_{V_i} r \, dx \, dy}{\int_{V_i} dx \, dy},$$

If $g_i = c_i$ - centroidal Voronoi tessellation (CVT)

Distance between generators and centroids - measure of mesh smoothness
Voronoi Tessellation - Lloyd’s Algorithm

1) Set positions of generators: \( g_i^0 : i = 1, \ldots, n \)
2) Construct Voronoi cells \( V_i^0 \) corresponding to \( g_i^0 \)
3) Define new positions of generators to be centroids of \( V_i^0 \): \( g_i^1 = c(V_i^0) \)
4) Repeat starting with \( #1 \) till some convergence criterion is satisfied

Mesh and Volume

Left - initial, central - five iterations, right - ”final”

Calculations by M. Kucharik

Lloyd’s Iteration - Mesh Smoothing
Reconnection-based Rezone Strategy

Requirements - Close to Lagrangian, Smooth Mesh, Continuity in Time

• Initial mesh at $t = 0$ is Voronoi mesh

• Voronoi mesh correspond to some generators - one generator per Voronoi cell

• Location of generators control the mesh, including connectivity

• Lagrangian Phase - There is no equation for movement of the generators

• On rezone stage we define new (rezoned) positions of generators which gives us new rezoned mesh - Voronoi mesh corresponding to new positions of generators.

• Rezone strategy is to how move generators
Algorithm for Movement of Generators

- Compute centroid of Lagrangian cell $\tilde{V}_i^{n+1}$ at $t^{n+1}$
  \[ c_i^{n+1} = \int_{\tilde{V}_i^{n+1}} r \, dx \, dy / \int_{\tilde{V}_i^{n+1}} dx \, dy \]

- "Lagrangian" Movement of Generators
  \[ g_i^{n+1,Lag} = g_i + \Delta t \bar{u}_i, \quad \bar{u}_i = \frac{1}{|P(c_i^n)|} \sum_{p \in P(c_i^n)} u_p \]

  - $g_i^{n+1,Lag}$ - inside Lagrangian cell at $t^{n+1}$

- Final position of generators
  \[ g_i^{n+1} = g_i^{n+1,Lag} + \omega_i \left[ (c)^{n+1} - g_i^{n+1,Lag} \right] \quad \omega_i \in [0, 1] \]

  - Generator at $t^{n+1}$ lies between Lagrangian position and position of centroid of Lagrangian cell at $t^{n+1}$
  - $\omega_i = 0$ - Lagrangian position; $\omega_i = 1$ - centroid of new Lagrangian cell - one iteration of Lloyd’s - smoothing the mesh
  - Choice of $\omega$ - Uniform Translation or Solid Rotation $\omega_i = 0$
Computation of the $\omega_i$

• The principle of **material frame indifference**: uniform translation or rotation
  $$\omega_i = 0$$

• Deformation gradient tensor $F$
  $$F = \begin{pmatrix} \frac{\partial X^{n+1}}{\partial X^n} & \frac{\partial X^{n+1}}{\partial Y^n} \\ \frac{\partial Y^{n+1}}{\partial X^n} & \frac{\partial Y^{n+1}}{\partial Y^n} \end{pmatrix}$$

  Jacobian matrix of the map that connects the Lagrangian configurations of the flow at time $t^n$ and $t^{n+1}$.

• The right Cauchy-Green strain tensor
  $$C = F^t F$$
  
  o **C is a 2 × 2 symmetric positive definite tensor**
  o **Reduces to the unitary tensor in case of uniform translation or rotation**
  o **Two positive eigenvalues**: $\lambda_1 \leq \lambda_2$ - the rates of dilation

• Definition of $\omega_i$
  $$\omega_i = \frac{1 - \frac{\lambda_{1,i}}{\lambda_{2,i}}}{1 - \min_i \frac{\lambda_{1,i}}{\lambda_{2,i}}}$$
Vortex Formation

\[ \gamma_1 = 1.5 \]
\[ \rho_1 = 1 \]
\[ P_1 = 1 \]
\[ \gamma_3 = 1.5 \]
\[ \rho_3 = 0.125 \]
\[ P_3 = 0.1 \]
\[ \gamma_2 = 1.4 \]
\[ \rho_2 = 1 \]
\[ P_2 = 0.1 \]

Time 2.7 - just before Lagrangian calculation stops because of mesh tangling - mesh and density.

Left - Lagrangian, Center - Standard ALE (Con), Right - ReALE (Con)
Vortex Formation

Time 2.72 - when Lagrangian calculation stops because of mesh tangling - coloring by initial region.

Left - Lagrangian, Center - Standard ALE (Con), Right - ReALE (Con)

Shows how Lagrangian is movement of the mesh
Vortex Formation

Left - Eulerian, Right - Standard ALE (Con)

Left - ReALE (Con), Right - ReALE (VOF)

Final time moment - 3.3
Vortex Formation

Left - ReALE (Con) density, mesh; Right - ReALE (Con) - trace $c (1 - c)$

Left - ReALE (VOF) density, mesh; Right - ReALE (VOF) - Materials

Final time moment - 3.3
The Interaction between a Planar Shock Wave and a Square Cavity

O. Igra, J. Falcovitz, H. Reichenbach and W. Heilig

"Experimental and Numerical Study of the Interaction between a Planar Shock Wave and a Square Cavity"

The Interaction between a Planar Shock Wave and a Square Cavity

Igra et al.

ReALE
The Interaction between a Planar Shock Wave and a Square Cavity

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Figure 9. As in figure 3 but at $t = 200 \mu s$.

Igra et al. ReALE
The Interaction between a Planar Shock Wave and a Square Cavity

The interaction between a planar shock wave and a square cavity

Figure 12. As in figure 3 but at $t = 340 \mu s$. 

Igra et al.  

ReALE
Conclusion

Summary

• New Reconnection-Based ALE - ReALE Method
• Demonstrated Performance on Test Problems
• On Test Examples ReALE Performs Better than Standard ALE
• NO USER INTERVENTION

More Information

• webpage: cnls.lanl.gov/~shashkov
• R. Loubère, P.-H. Maire, M. Shashkov, J. Breil, S. Galera
  *ReALE: A Reconnection-based Arbitrary-Lagrangian-Eulerian Method*,

• R. Loubère, P.-H. Maire, M. Shashkov
  *ReALE: A Reconnection Arbitrary-Lagrangian-Eulerian Method in Cylindrical Geometry*,