ReALE - Reconnection-Based ALE

Mikhail J. Shashkov

XCP-4, Methods & Algorithms,
Los Alamos National Laboratory,
Los Alamos, New Mexico, 87545, USA

e-mail:shashkov@lanl.gov; webpage: cnls.lanl.gov/∼ shashkov

Raphaël Loubére - CNRS at Université de Toulouse, France
Pierre-Henri Maire - CEA-CESTA BP 2, France
Jérôme Breil - CELIA, Université de Bordeaux, France
Stéphane Galera - INRIA, Team Bacchus, France

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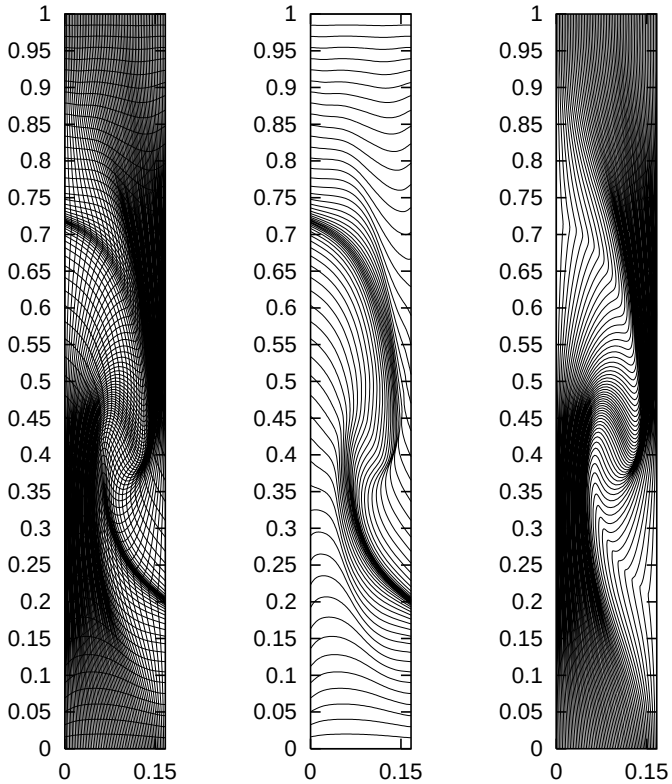
Arbitrary Lagrangian-Eulerian (ALE) Methods

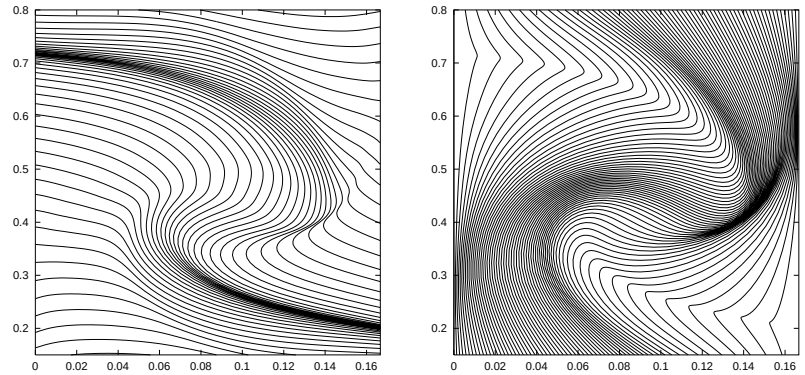
- **Explicit Lagrangian (solving Lagrangian equations) phase — grid is moving with fluid**

- **Rezone phase — changing the mesh (improving geometrical quality, smoothing, adaptation) — mesh movement**

- **Remap phase (conservative interpolation) — remapping flow parameters from Lagrangian grid to rezoned mesh**

Rayleigh-Taylor Instability — Limitation of ALE





Reference-Jacobian Rezone Strategy

(Knupp, Margolin, Shashkov)

Solution: ReALE - Reconnection-based ALE

Rezone phase — Mesh is allowed to change connectivity

ReALE — Reconnection-based ALE

- **Lagrangian phase — General polygonal meshes**
- **Rezone phase — Allows mesh reconnection**
- **Remap phase — Remapping from one polygonal mesh to another**

The Devil is in the Details

Multimaterial Lagrangian Hydrodynamics

- **Method of Concentrations**

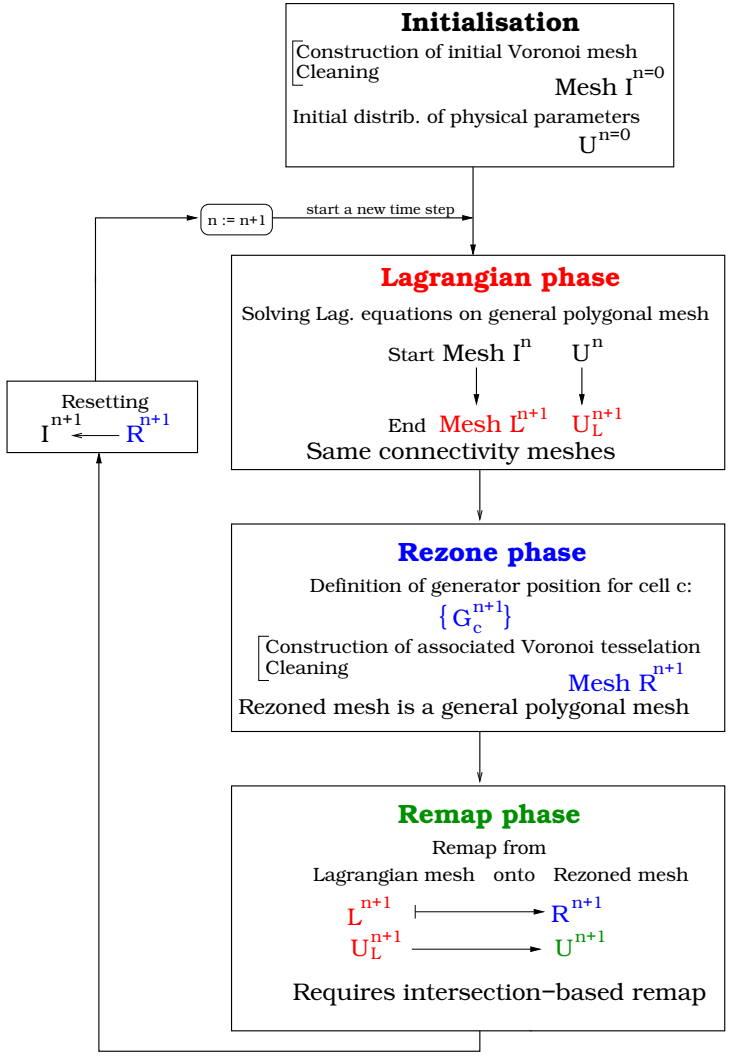
  *** Transport equations for evolving concentrations, rescaling.**

  *** Mixture assumption - iso-pressure, iso-temperature, leads to effective γ for mixture of ideal gasses**

- **Volume-of-fluid (VOF); Moment-of-Fluid (MOF)**

  - **Interface Reconstruction**
  - **Closure models for mixed cells**

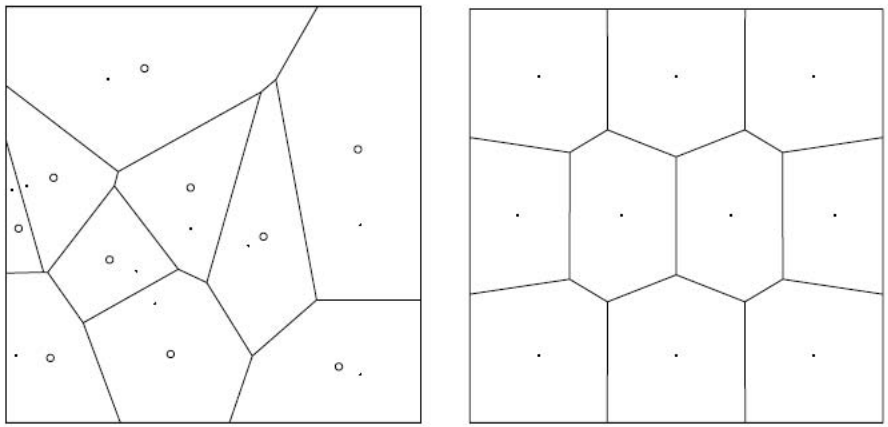
ReALE - Flowchart



Voronoi Tessellation - Definition

Set of generators: $\mathbf{g}_i = (x_i, y_i)$
Voronoi cell: $V_i = \{\mathbf{r} = (x, y) : |\mathbf{r} - \mathbf{g}_i| < |\mathbf{r} - \mathbf{g}_j|\,,$ **for all** $j \neq i\}$
Centroid of the cell

$\mathbf{c}_i = \int_{V_i} \mathbf{r}\, dx\, dy / \int_{V_i} dx\, dy$**,**
If $\mathbf{g}_i = \mathbf{c}_i$ **-centroidal Voronoi tessellation (CVT)**

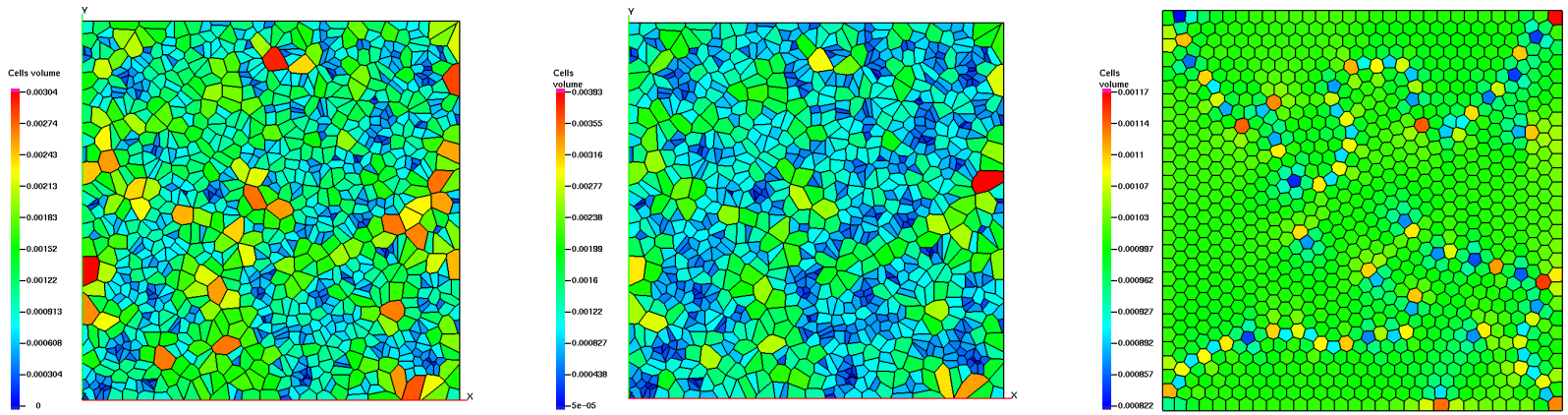


Left - generators, Voronoi cells, centroids; Right - centroidal Voronoi Tessellation

Distance between generators and centroids - measure of mesh smoothness

Voronoi Tessellation - Lloyd's Algorithm

1) **Set positions of generators:** $\mathbf{g}_i^0 \ : i = 1, \ldots, n$
2) **Construct Voronoi cells** V_i^0 **corresponding to** $\mathbf{g}_i^0$
3) **Define new positions of generators to be centroids of** V_i^0**:** $\mathbf{g}_i^1 = \mathbf{c}(V_i^0)$
4) **Repeat starting with** $\#1$ **till some convergence criterion is satisfied**



Mesh and Volume

Left - initial, central - five iterations, right - "final"

Calculations by M. Kucharik

Lloyd's Iteration - Mesh Smoothing

Reconnection-based Rezone Strategy

Requirements - Close to Lagrangian, Smooth Mesh, Continuity in Time

- **Initial mesh at $t = 0$ is Voronoi mesh**
- **Voronoi mesh correspond to some generators - one generator per Voronoi cell**
- **Location of generators control the mesh, including connectivity**
- **Lagrangian Phase - There is no equation for movement of the generators**
- **On rezone stage we define new (rezoned) positions of generators which gives us new rezoned mesh - Voronoi mesh corresponding to new positions of generators.**
- **Rezone strategy is to how move generators**

Algorithm for Movement of Generators

- **Compute centroid of Lagrangian cell $\tilde{V}_i^{n+1}$ at t^{n+1}**
  $\mathbf{c}_i^{n+1} = \int_{\tilde{V}_i^{n+1}} \mathbf{r}\ dx\, dy / \int_{\tilde{V}_i^{n+1}}\ dx\, dy$
- **"Lagrangian" Movement of Generators**

$$\mathbf{g}_i^{n+1,Lag} = \mathbf{g}_i + \Delta t\, \bar{\mathbf{u}}_i\,, \qquad \bar{\mathbf{u}}_i = \frac{1}{|\mathcal{P}(c_i^n)|} \sum_{p \in \mathcal{P}(c_i^n)} \mathbf{u}_p$$

  - $\mathbf{g}_i^{n+1,Lag}$ **- inside Lagrangian cell at t^{n+1}**
- **Final position of generators**

$$\mathbf{g}_i^{n+1} = \mathbf{g}_i^{n+1,Lag} + \omega_i \left[(c)^{n+1} - \mathbf{g}_i^{n+1,Lag} \right] \qquad \omega_i \in [0, 1]$$

  - **Generator at t^{n+1} lies between Lagrangian position and position of centroid of Lagrangian cell ate t^{n+1}**
  - $\omega_i = 0$ **- Lagrangian position;** $\omega_i = 1$ **- centroid of new Lagrangian cell - one iteration of Lloyd's - smoothing the mesh**
  - **Choice of ω - Uniform Translation or Solid Rotation** $\omega_i = 0$

Computation of the ω_i

- **The principle of material frame indifference: uniform translation or rotation** $\omega_i = 0$
- **Deformation gradient tensor** F

$$\mathsf{F} = \begin{pmatrix} \frac{\partial X^{n+1}}{\partial X^n} & \frac{\partial X^{n+1}}{\partial Y^n} \\ \frac{\partial Y^{n+1}}{\partial X^n} & \frac{\partial Y^{n+1}}{\partial Y^n} \end{pmatrix},$$

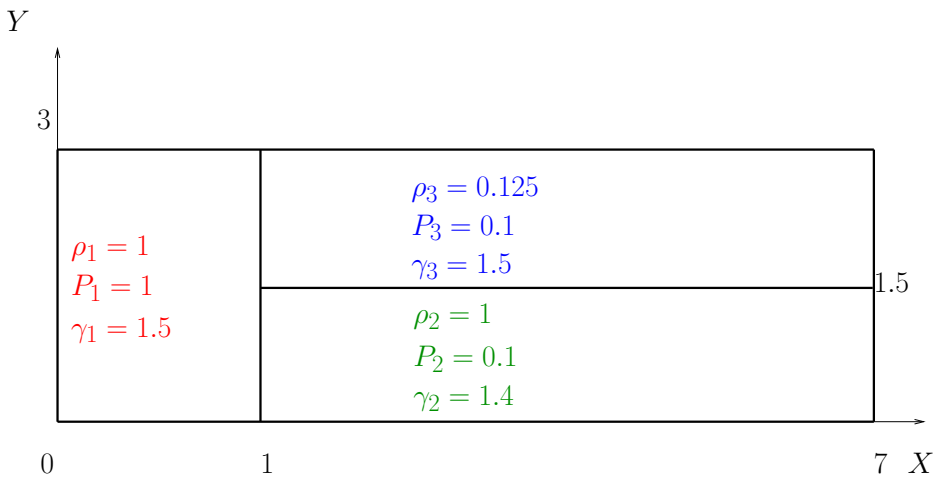
  Jacobian matrix of the map that connects the Lagrangian configurations of the flow at time t^n and t^{n+1}.
- **The right Cauchy-Green strain tensor**

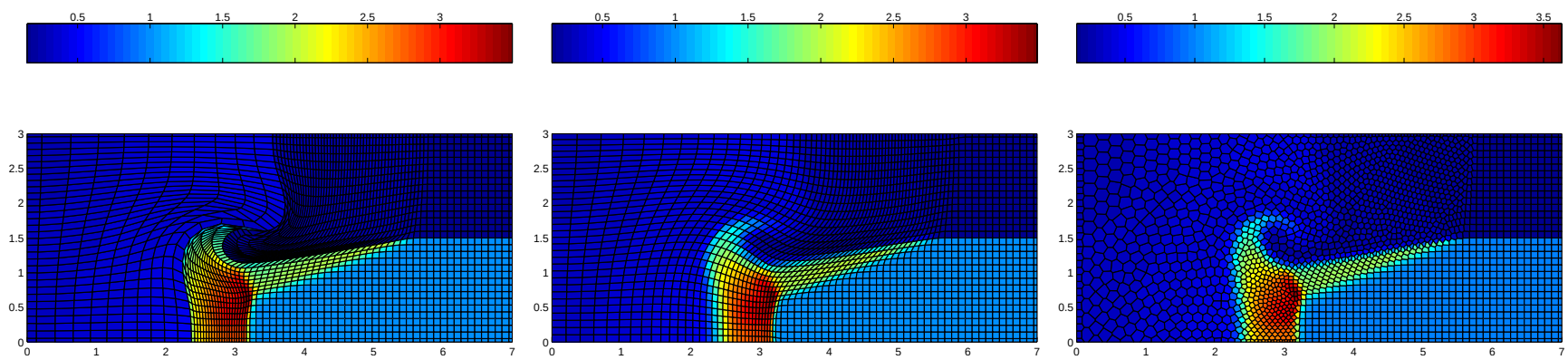
$$\mathsf{C} = \mathsf{F}^t \mathsf{F}$$

  - C **is a** 2×2 **symmetric positive definite tensor**
  - **Reduces to the unitary tensor in case of uniform translation or rotation**
  - **Two positive eigenvalues:** $\lambda_1 \leq \lambda_2$ **- the rates of dilation**
- **Definition of** ω_i

$$\omega_i = \left(1 - \frac{\lambda_{1,i}}{\lambda_{2,i}}\right) \Big/ \left(1 - \min_i \frac{\lambda_{1,i}}{\lambda_{2,i}}\right)$$

Vortex Formation

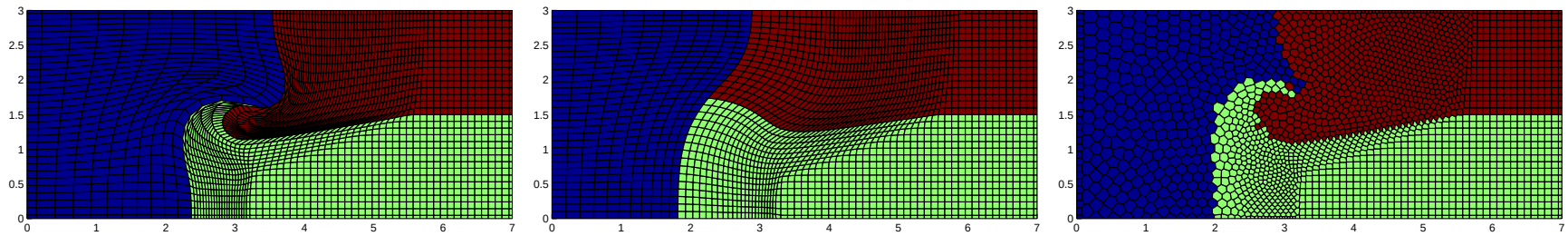




Time 2.7 - just before Lagrangian calculation stops because of mesh tangling - mesh and density.

Left - Lagrangian, Center - Standard ALE (Con), Right - ReALE (Con)

Vortex Formation

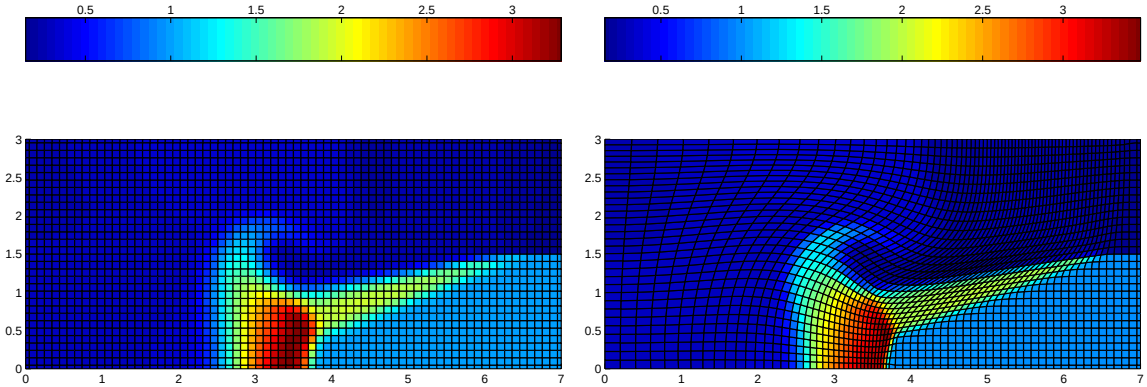


Time 2.72 - when Lagrangian calculation stops because of mesh tangling - coloring by initial region.

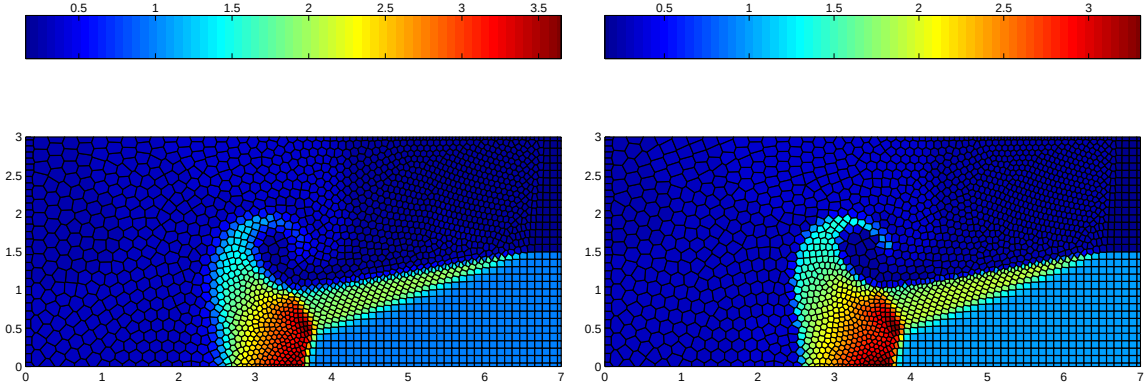
Left - Lagrangian, Center - Standard ALE (Con), Right - ReALE (Con)

Shows how Lagrangian is movement of the mesh

Vortex Formation



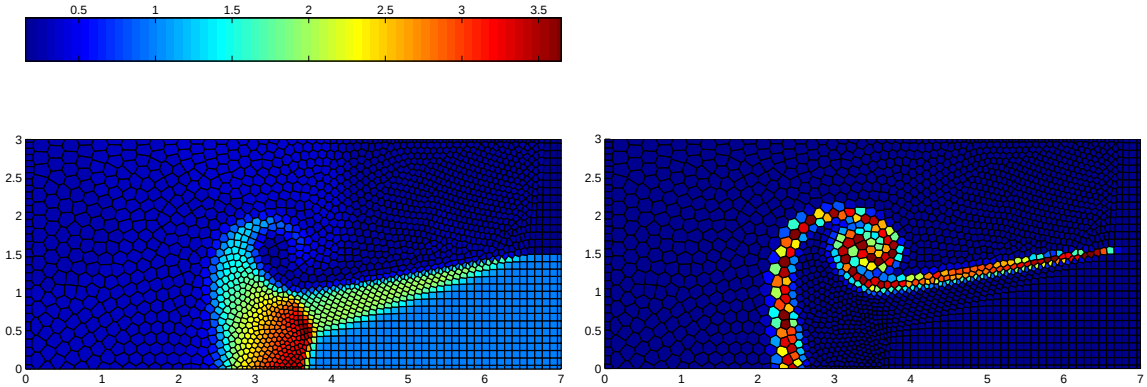
Left - Eulerian, Right - Standard ALE (Con)



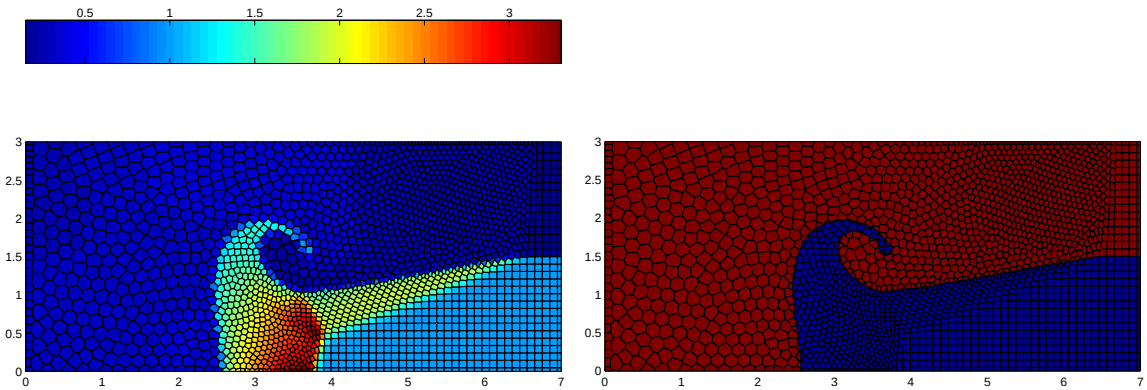
Left - ReALE (Con), Right - ReALE (VOF)

Final time moment - 3.3

Vortex Formation



Left - ReALE (Con) density,mesh; Right - ReALE (Con) - trace $c\,(1-c)$



Left - ReALE (VOF) density,mesh; Right - ReALE (VOF) - Materials

Final time moment - 3.3

The Interaction between a Planar Shock Wave and a Square Cavity

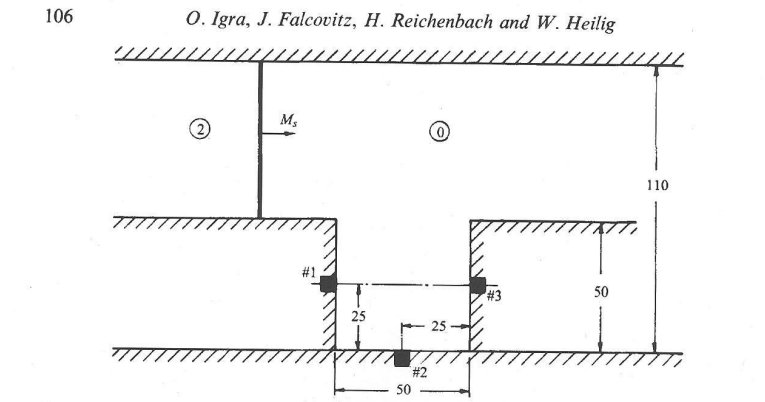
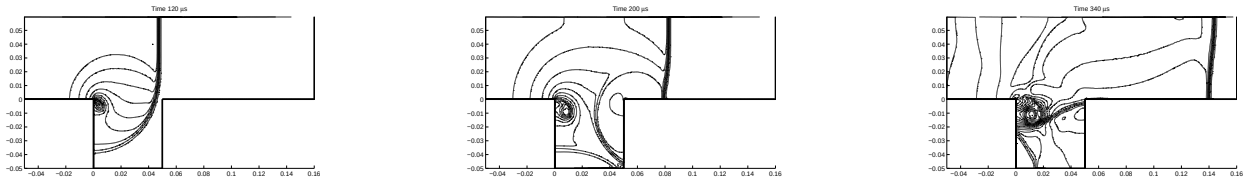


FIGURE 1. Schematic description of the flow field considered prior to the arrival of the incident shock wave at the cavity. All dimensions are in mm. Solid squares show pressure transducers, labelled nos 1–3. The computation domain is a rectangle 210 × 110 mm, starting 50 mm to the left of the cavity.



O. Igra, J. Falcovitz, H. Reichenbach and W. Heilig

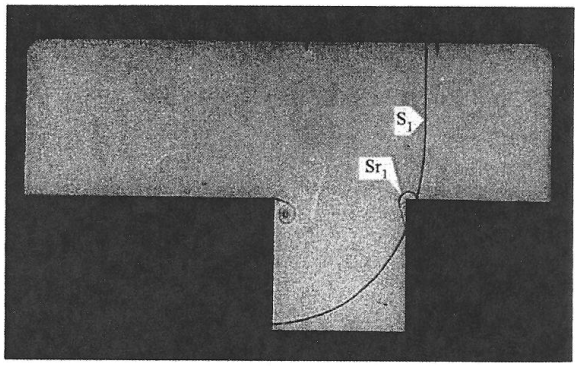
"Experimental and Numerical Study of the Interaction between a Planar Scock Wave and a Square Cavity"

J. Fluid Mech. (1996) v. 313. pp. 105-130

The Interaction between a Planar Shock Wave and a Square Cavity

The interaction between a planar shock wave and a square cavity

(*a*)



(*b*)

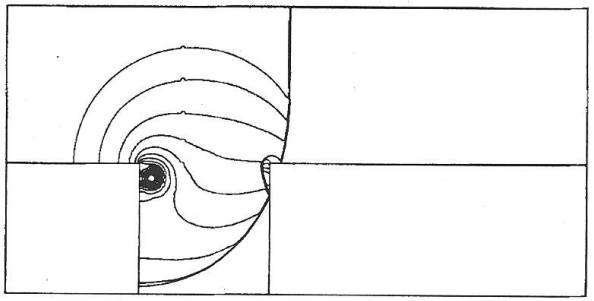
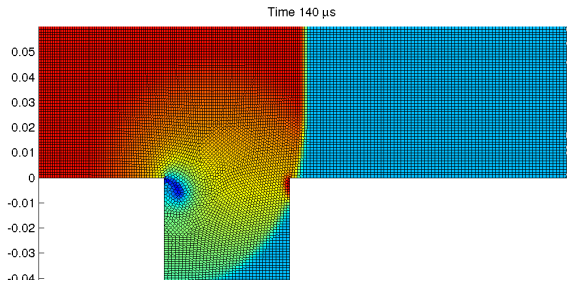
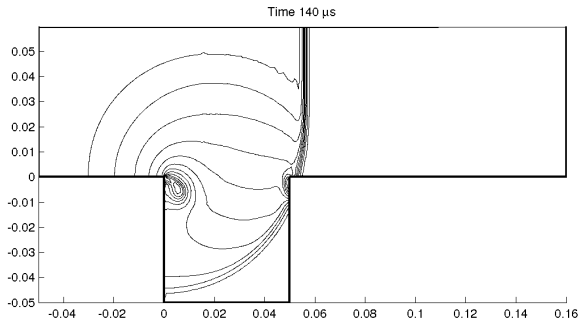


FIGURE 6. As in figure 3 but at $t = 140\ \mu s$.





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The Interaction between a Planar Shock Wave and a Square Cavity

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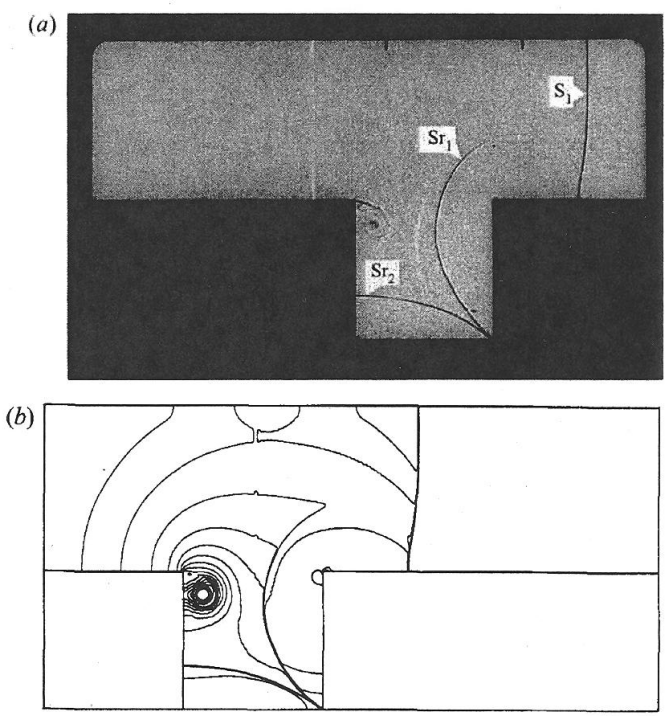
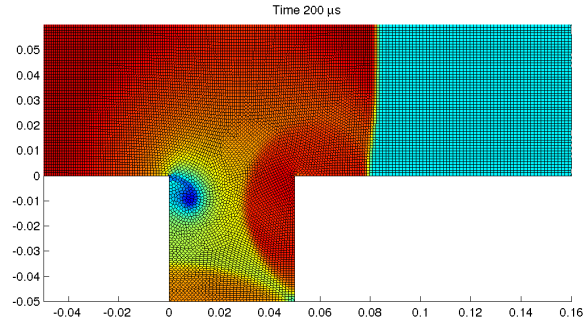
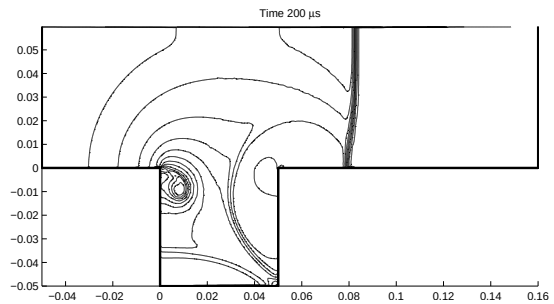


FIGURE 9. As in figure 3 but at $t = 200$ μs.





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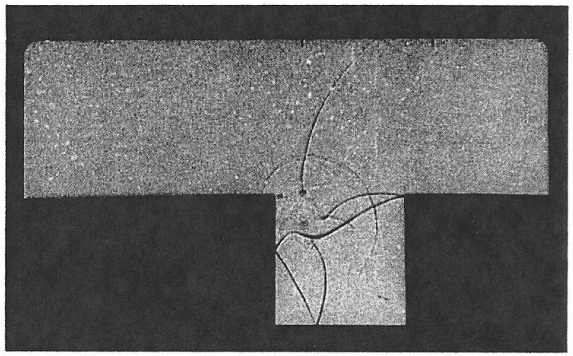
The Interaction between a Planar Shock Wave and a Square Cavity

The interaction between a planar shock wave and a square cavity

(*a*)



(*b*)

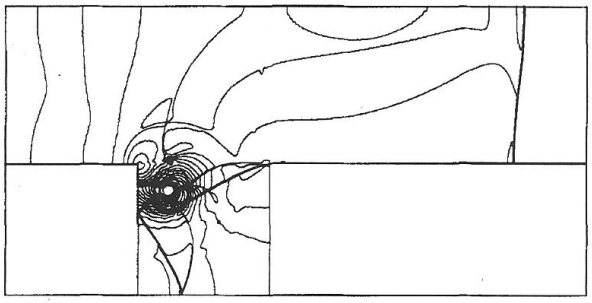
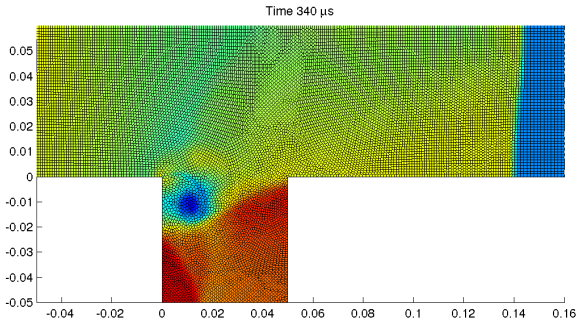
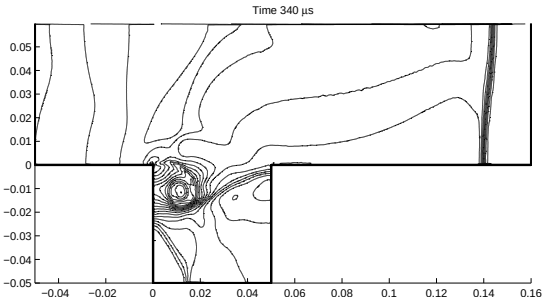


FIGURE 12. As in figure 3 but at $t = 340$ μs.





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ReALE

Conclusion

Summary

- New Reconnection-Based ALE - ReALE Method
- Demonstrated Performance on Test Problems
- On Test Examples ReALE Performs Better than Standard ALE
- NO USER INTERVENTION

More Information

- webpage: cnls.lanl.gov/$\sim$ shashkov
- R. Loubère, P.-H. Maire, M. Shashkov, J. Breil, S. Galera
  ReALE: A Reconnection-based Arbitrary-Lagrangian-Eulerian Method, Journal of Computational Physics Volume 229 , Issue 12 (June 2010), Pages: 4724-4761.
- R. Loubère, P.-H. Maire, M. Shashkov
  ReALE: A Reconnection Arbitrary-Lagrangian-Eulerian Method in Cylindrical Geometry, Computers & Fluids, Volume 46, Issue 12 (July 2011), Pages: 59-69.