Nonlinear Optimization

$$\begin{align*}
(P) \quad \text{minimize} & \quad f(x) \\
\text{subject to} & \quad c(x) \geq 0
\end{align*}$$

- Inequalities make $P$ challenging: combinatorial ...
- Application: power grid, core-reloading, ...

Why Another Solver???

xTiNO is not a solver, but a toolkit ...
- There is no single "best" method for nonlinear optimization.
- Implement range of methods within single framework: tailor solver for mixed-integer, control, complementarity, ...
- Separate linear algebra layer from optimization.
- Ready solvers for emerging architectures.

Trust-Region Framework

$$(M(x_k)) \text{ local model of } (P) \text{ around iterate } x_k$$

Given $x_k$, set $k = 0$;

while $x_k$ is not optimal do
  $\hat{x}$ ← solve local model $(M(x_k))$
in trust-region $\|x - x_k\| \leq \Delta_k$;
  if $\hat{x}$ better than $x$ then
    $x_{k+1} = \hat{x}$ increase $\Delta_{k+1} = 2\Delta_k$
  else
    $x_{k+1} = x_k$ decrease $\Delta_{k+1} = \Delta_k/2$
end
end

Sequential Quadratic Programming (SQP)

$$\begin{align*}
(M(x_k)) \quad \text{minimize} & \quad f_k + \nabla^T_k (x - x_k) + \frac{1}{2}(x - x_k)^T H_k (x - x_k) \\
\text{subject to} & \quad c_k + \nabla^T c_k (x - x_k) \geq 0
\end{align*}$$

- Fast local convergence, good warm-starts.
- Snag: QP solves are computationally expensive.

Sequential Linear Quadratic Programming (SLQP)

$$\begin{align*}
(M(x_k)) \quad \text{minimize} & \quad f_k + \nabla^T_k (x - x_k) \\
\text{subject to} & \quad c_k + \nabla^T c_k (x - x_k) \geq 0
\end{align*}$$

- Solve linear program to estimate active set

Comparing SQP-Like Methods

- FASTr: filter-SQP method
- FASTR-RLP: filter-RLP/EQP
- SNOPT: quasi-Newton SQP
- SLOP: filter-SLCP method

Filter & Funnel Methods

- Goal: accept progress based only on objective function!
- Filter methods (left) converge to feasible limit point.
- Funnel methods (right) filter with single entry: $(U, -\infty)$.

Comparing SLQP & SQP

- LP & linear solves are computationally cheaper.
- EQP ensures fast local convergence.
- Snag: LP/QP still hard to parallelize.

Nonmonotone Filter Methods

Nonmonotone filter:
- Accept $x_{k+1}$ if not dominated by more than $M > 0$ entries.
- Standard filter is equivalent to using $M = 0$.
- Nonmonotone filter avoids Maratos effect:
  - Switch between local/global filter.
  - Local filter: steps inside trust-region.
  - Global filter: promotes global convergence.
  - Reset local filter, whenever TR active.
  - Reset ⇒ flush old information from local filter.

Software Design

Open-source C++ framework
- Abstract classes for ...
- Problem description: user interface (AMPL, CUTEr).
- Algorithms and methods: SQP et al.
- Subproblem solvers: OP, LP, EQP, LCP, ...
- Matrices, vectors & linear algebra (see SQP).
- Code re-use: e.g. restoration phase same as optimality.
- External links to row solver classes:
  - Interior-point methods & cross-over.
  - Sequential LCP methods $\approx$ SOCP ... toward SOCP.

New Augmented Lagrangian Methods

$$L_s(x, s, y) := f(x) - y^T c(x) - s + \frac{1}{2} s^T L_s(x, s) ||c(x) - s||^2$$

where $s \geq 0$ slack variables ($c(x) - s = 0$).

- Approximately minimize augmented Lagrangian
  - "approx." minimize $L_s(x, s, y)$

- Solve EQP for fast convergence (see SLOP above).
- Advantages of augmented Lagrangian methods:
  - Both computational modules parallelize.
  - Fast local convergence.