Aspect-ratio effects in rotating, stably stratified Boussinesq flows

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Motivation

The Boussinesq approximation is a starting point to describe a fairly broad range of oceanic and atmospheric phenomena, depending on the choice of the parameters, aspect-ratio δ, rotation frequency f, and buoyancy frequency N. Despite the fact that geophysical flows are largely confined to small aspect-ratio domains, idealized numerical studies of the Boussinesq equations are a mix of calculations and performed in domains of unit aspect-ratio and small aspect-ratio. From this mix it is difficult to discern aspect ratio effects from other effects of physical and/or numerical choices such as external forcing, grid spacing and effective viscosity. We present data from two simulations with Burger number Bu = 1, with aspect ratios 1/4 and 1/16 respectively, and a third simulation with Bu = 4 and aspect ratio 1. The cases with Bu = 1 are of particular interest because the atmosphere in mid-latitudes is Bu = O(1). We take initial steps toward quantifying aspect-ratio effects by

(i) characterizing the vertical lengthscale that develops and

(ii) measuring the fraction of the energy in the wave modes of the small scales.

Definitions

- domain aspect-ratio δ = domain height/domain length = H/L
- f is twice rotation rate; N is Brunt-Vaisala (buoyancy) frequency
- velocity scale U is based on the forcing scale
- Rossby number Ro = U/(Lf)
- Froude number Fr = U/(HN)
- Burger number Bu = Ro/Fr = (NH)/(fL) = δ N/f

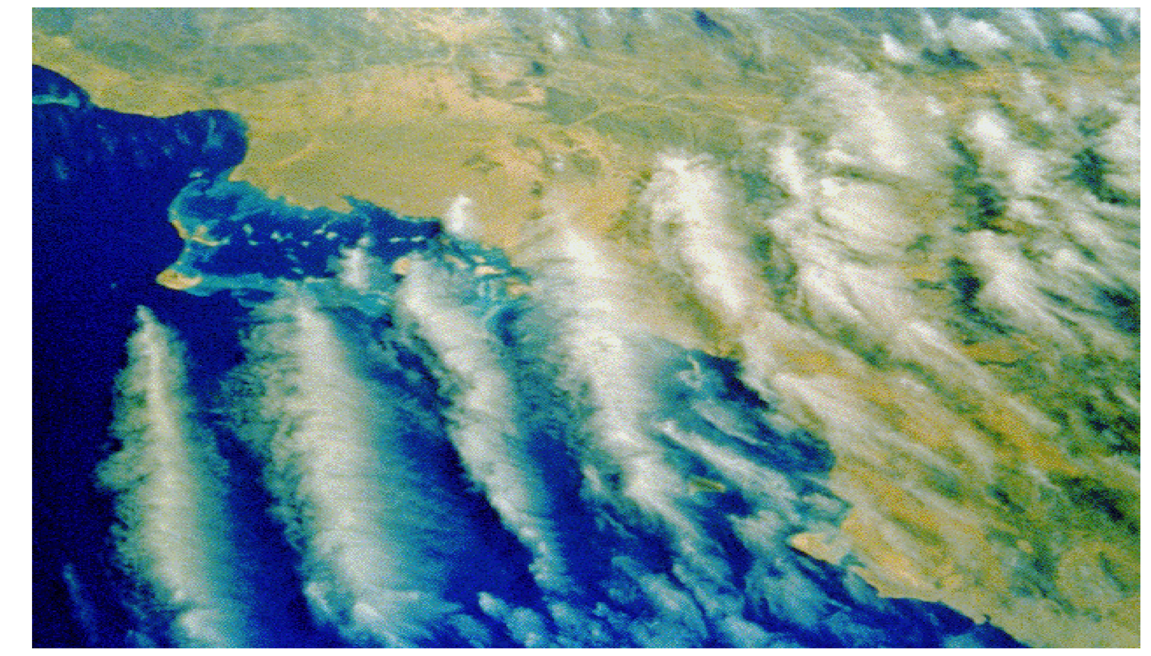
Simulation parameters

- Boussinesq equations computed on triply periodic domain with stable, constant vertical stratification and constant rotation about the vertical.
- pseudo spectral calculation
- all modes forced stochastically at scales ¼ the height of the domain
- Isotropic resolution of the smallest scales:
  - $(k_x)_{max} = (k_y)_{max} = (k_z)_{max}$
- comparison of three flows with varying δ and N/f
  - δ = 1, N/f = 4 (Ro = 0.0091, Fr = 0.0023, Bu = 4) 640 x 640 x 640 gridpoints
  - δ = 1/4, N/f = 4 (Ro = 0.002; Fr = 0.002; Bu = 1) 2048 x 2048 x 512 gridpoints (15M CPU hours)
  - δ = 1/16, N/f = 16 (Ro = 0.002, Fr = 0.002; Bu = 1) 2048 x 2048 x 128 gridpoints (10M CPU hours)

Conclusions

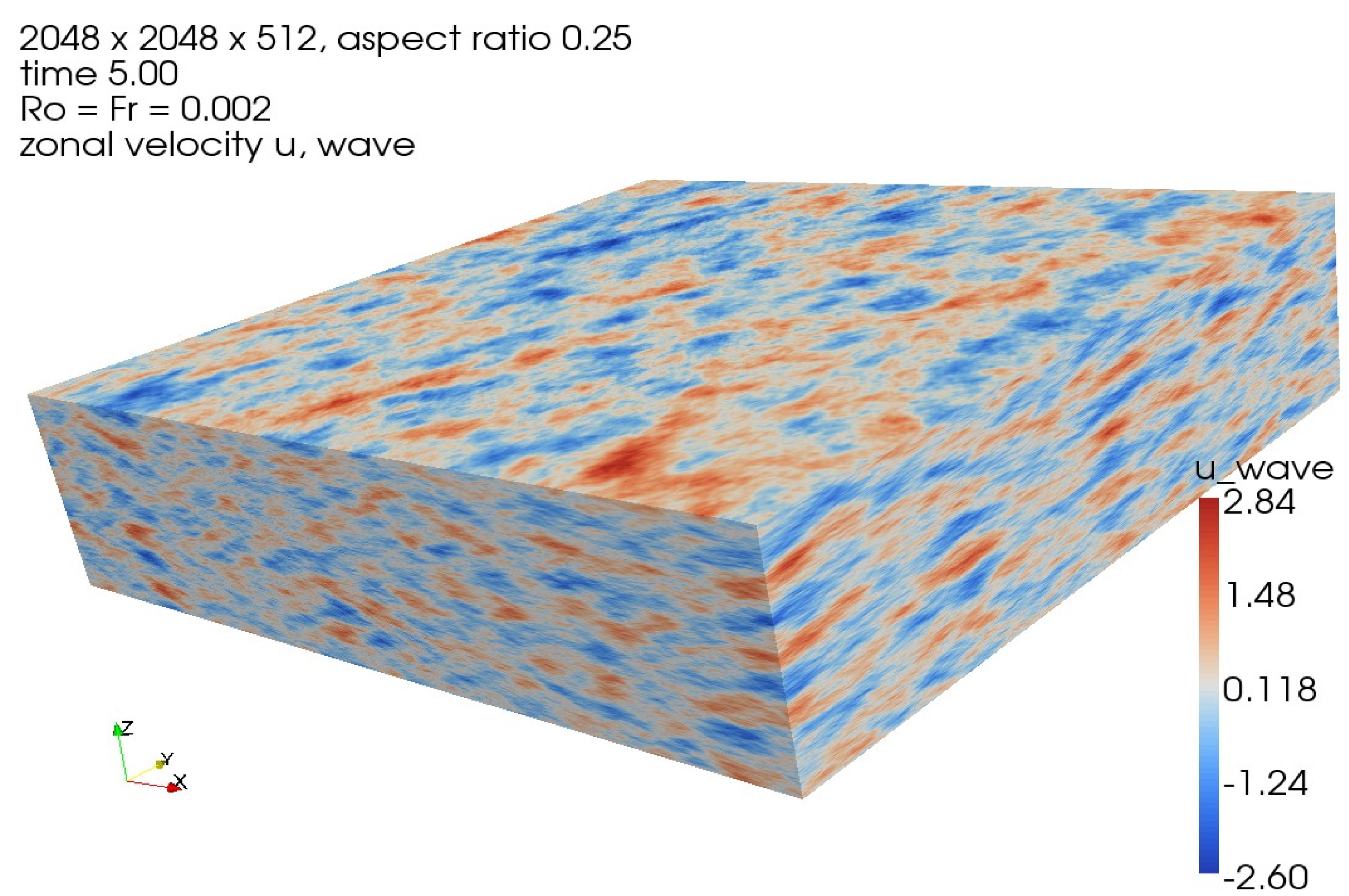
- For fixed Bu = 1 decreasing δ from 1/4 to 1/16 results in
  - an increase in relative thickness of the vertical scale from ~8% to ~17%
  - a decrease in the fraction of small-scale energy in the wave-modes from ~85% to ~40%
- For fixed N/f = 4 decreasing δ from 1 to 1/4 results in
  - an increase in relative thickness of the vertical scale from ~6% to ~8%
  - a decrease in the fraction of small-scale energy in the wave-modes from ~95% to ~85%.
- These are the first large-scale simulations to quantify these effects.

Roll clouds help visualize inertia-gravity waves in the jet stream over Saudi Arabia/Red Sea

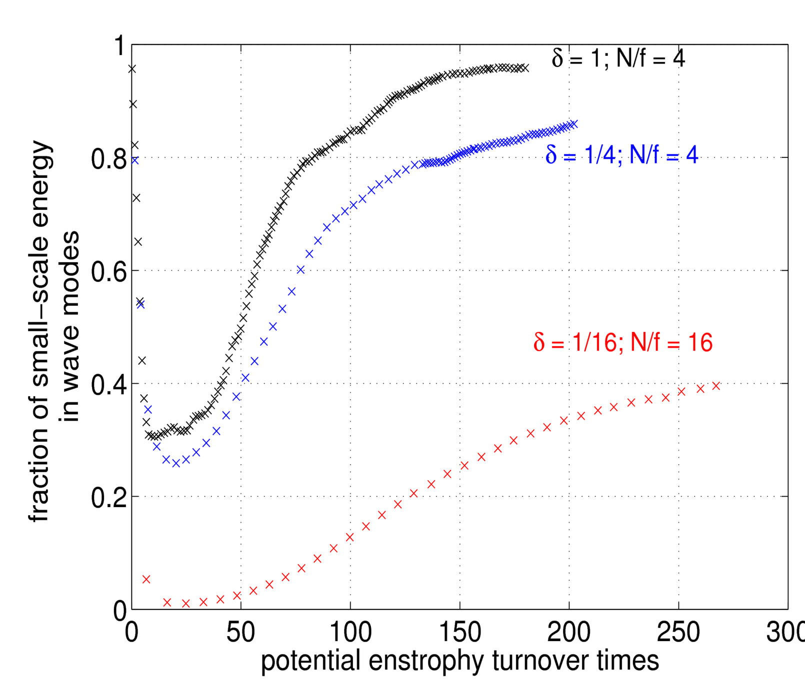


A larger motivation for fundamental studies of Boussinesq equations, which form the foundation for much of fluids modeling of the ocean/atmosphere/climate dynamics, is to understand the dynamical role of fast, small-scale inertia-gravity waves. The waves, which are often under-resolved or need to be modeled in *ad hoc* ways, can feed energy into large scales to produce $O(1)$ effects. The images on the left show how the large scale jet stream and the small scale inertia-gravity waves 'riding' on it together form a complex geophysical flow.

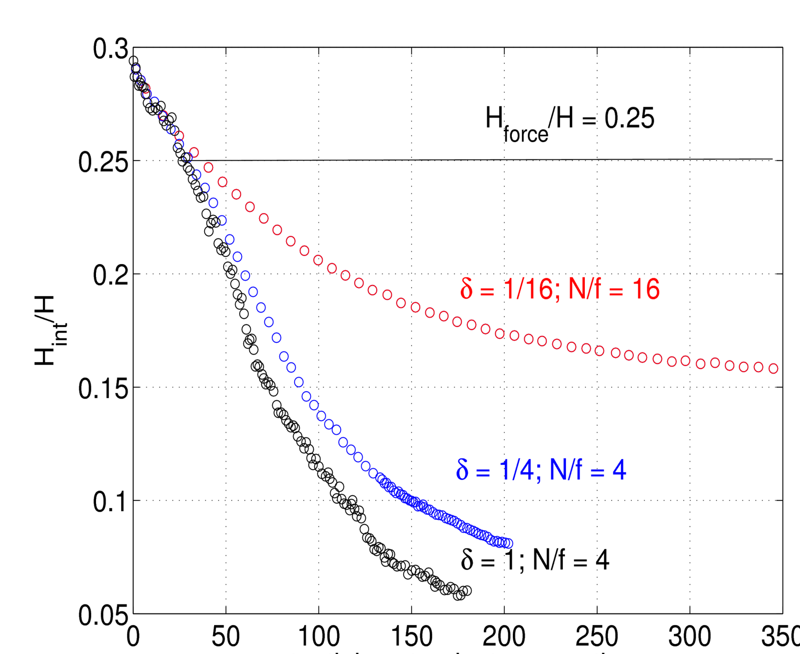
Layers in wave component of horizontal velocity in domain with δ = 1/4



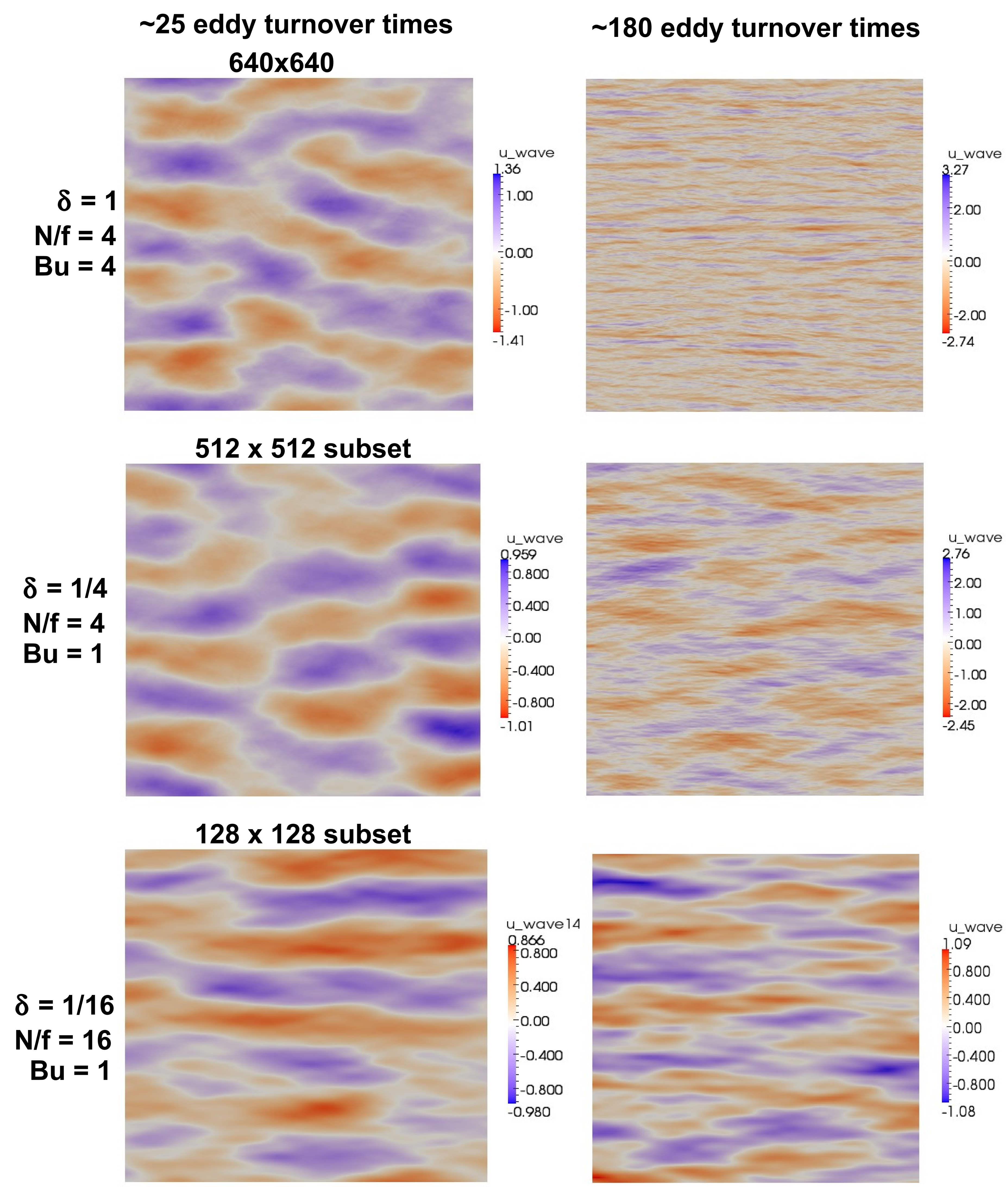
Relative fraction of small-scale energy in wave modes



Evolution of internal vertical length-scale



High-pass filtered wave component of horizontal velocity



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