Coarse-graining the dynamics of (and on) evolving graphs:
Algorithms and Computation.
Yannis Kevrekidis, students and collaborators (Princeton University)

Dynamical models and coarse-graining

Dynamical models of networks
- Usually presented in terms of nodes and edges
- Macroscopic "system-level" description is usually NOT available
- OBJECTIVE: Find coarse models (macroscopic descriptions) for the evolutionary network problems.

Equation-free modeling for coarse-graining multiscalar systems
- The key step is to define suitable coarse variables (observables);
- e.g., the network degree distribution.
- Then, lifting and restriction operators are constructed to translate between fine and coarse states as shown below.

A random evolution model of networks
- Dynamics at each time step:
  - Choose 2 random nodes and connect them if not already connected
  - Choose an edge with probability p

Simulation:
- 100 networks with 100 nodes each
- p = 0.8
- Degree distribution evolves smoothly
- But, do we need other statistics (like triangles)?

Direct simulation with the same degree distribution
- but two different distributions of clustering coefficients

Kuramoto model on a static network

Synchronization of coupled oscillators on a network
- General coupled oscillator model:
  \[ \dot{\theta}_i = \omega_i + \sum_{j \neq i} K \sin(\theta_j - \theta_i) \]
- Phases, \( \theta_i \) of oscillators
- Heterogeneous frequencies, \( \omega_i \)
- Kuramoto-model on a network:
  \[ K \sum_{j \neq i} \sin(\theta_j - \theta_i) \]
- A is the adjacency matrix of the network:
  \[ A_{ij} = 1 \text{ if there is a link between nodes } i \text{ and } j \]
- K is the coupling strength
- Networks constructed to facilitate separation of timescales

**Definition:** Order parameter (r) as
\[ r = \frac{1}{N} \sum_{i=1}^{N} \sin(\theta_i) \]

Using the Graph Laplacian

Normalized Laplacian graph Laplacian of the network (A)

S50 oscillators x 10 x 50 network

K = 0.5

Average state

Dynamical models and coarse-graining

K is the coupling strength
A is the adjacency matrix of the network;

Equation-free coarse-graining for coarse-graining multi-scale systems

Then, OBJECTIVE: Find coarse models (macroscopic descriptions) for the evolutionary network problems.

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Coarse integration, coarse fixed points

Heterogeneity and the coarse-grained model

Opinion propagation on a static network

Every person has an emotional state, \( x_i \in [-1,1] \)

Social network provides the private information

Development of correlations

Polynomial chaos

A new illustration: SIR as a heterogeneous social network

EQUATION-FREE AND VARIABLE-FREE MODELING FOR COMPLEX/MULTISCALED SYSTEMS: Coarse-grained computation in coarse and engineering using the coarse-grained models, B. E. Coifman, Yale, co-R