High-Order, Mapped-Multiblock, Finite-Volume Discretization of Gyrokinetic Systems Near the X Point of a Diverted Tokamak Geometry

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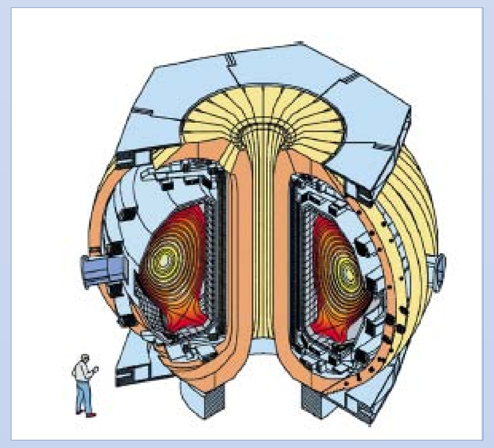
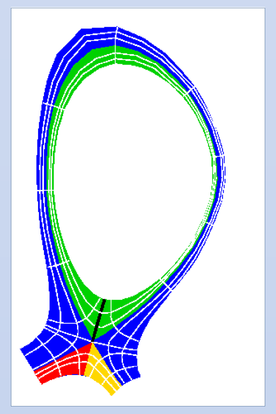
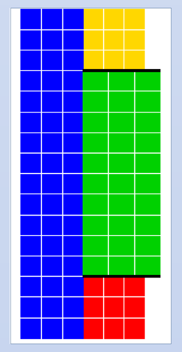
Simulation of tokamak edge plasmas requires the solution of PDEs in a geometry defined by magnetic flux surfaces. To better accommodate anisotropy, there is strong motivation for the use of mapped multiblock coordinates aligned with the flux surfaces:





Mapping



Field line following coordinate

Flux surface coordinate

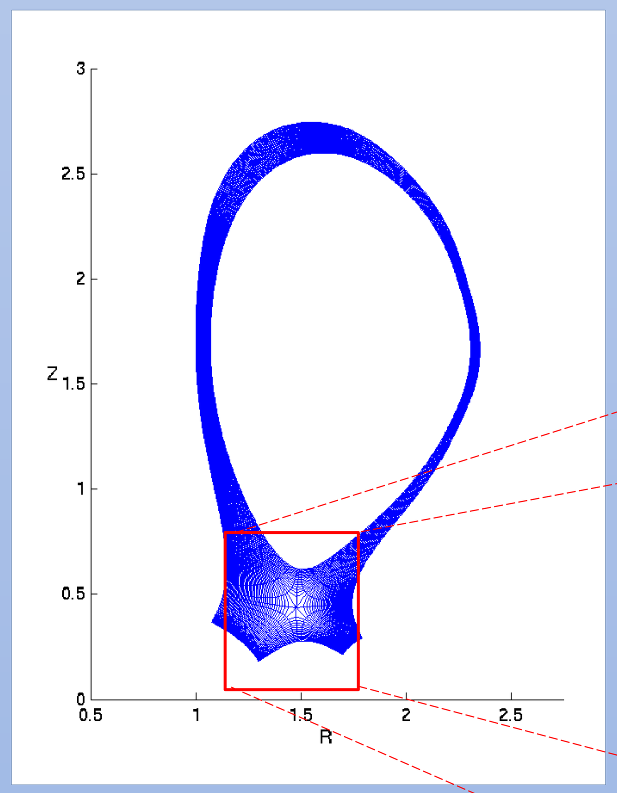
In previous work, we have developed a general formalism for the creation of high-order finite-volume discretizations in mapped coordinates:

$$\int_{\mathbf{X}(V_i)} \nabla_{\mathbf{x}} \cdot \mathbf{F} d\mathbf{x} = \sum_{\pm=+,-} \sum_{d=1}^{D} \pm \int_{A_d^{\pm}} \left(\mathbf{N}^T \mathbf{F}\right)_d d\mathbf{A}_{\xi} = h^{D-1} \sum_{\pm=+,-} \sum_{d=1}^{D} \pm F^d_{\mathbf{i}\pm\frac{1}{2}\mathbf{e}^d} + O(h^4)$$

where $$F^d_{\mathbf{i}+\frac{1}{2}\mathbf{e}^d} \equiv \sum_{s=1}^{D} \left\langle N_d^s \right\rangle_{\mathbf{i}+\frac{1}{2}\mathbf{e}^d} \left\langle F^s \right\rangle_{\mathbf{i}+\frac{1}{2}\mathbf{e}^d} + \frac{h^2}{12} \sum_{s=1}^{D} \left(\mathbf{G}_0^{\perp,d} \left(\left\langle N_d^s \right\rangle_{\mathbf{i}+\frac{1}{2}\mathbf{e}^d} \right) \right) \cdot \left(\mathbf{G}_0^{\perp,d} \left(\left\langle F^s \right\rangle_{\mathbf{i}+\frac{1}{2}\mathbf{e}^d} \right) \right)$$

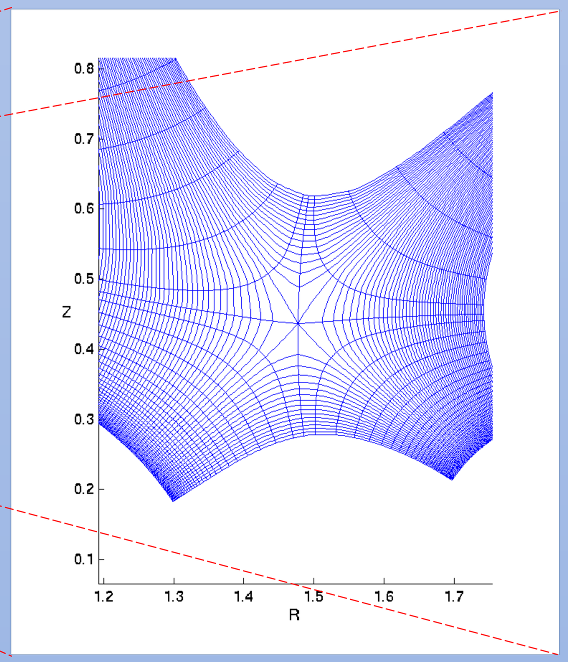
$\mathbf{G}_0^{\perp,d}$ = second-order accurate centered difference of $\nabla_{\xi} - \mathbf{e}^d \frac{\partial}{\partial \xi_d}$ and $\left\langle q \right\rangle_{\mathbf{i}+\frac{1}{2}\mathbf{e}^d} \equiv \frac{1}{h^{D-1}} \int_{A_d} q(\xi) d\mathbf{A}_{\xi} + O(h^4)$

These discretizations have been applied to a system of gyrokinetic equations in the development of COGENT (COntinuum Gyrokinetic Edge New Technology) using Chombo.

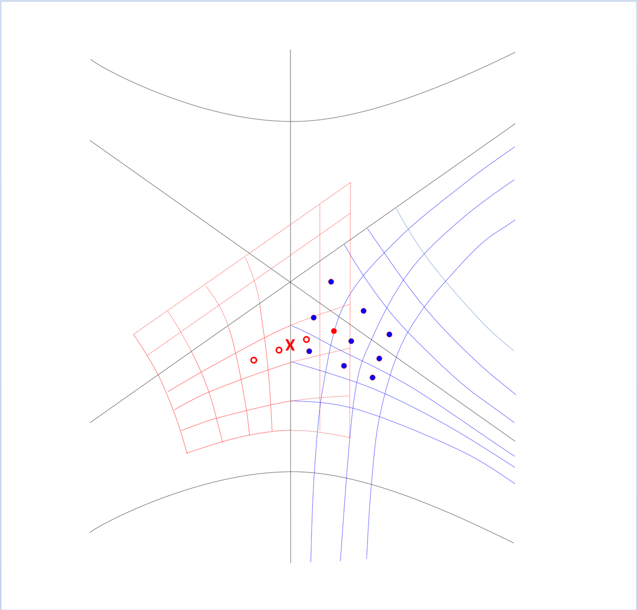


Original grid

Since the poloidal component of the magnetic field vanishes at the "X point", there is no need to be field aligned there. In fact, there is a penalty for any attempt to do so resulting from large derivatives corresponding to increasingly kinked field projections.



Approach: Modify the mappings in a neighborhood of the X point and use a least squares interpolation algorithm that provides accurate coupling across block boundaries:



- To find the cell average of f in a neighbor block ghost cell (centered at the red dot), assume a polynomial around the cell center:

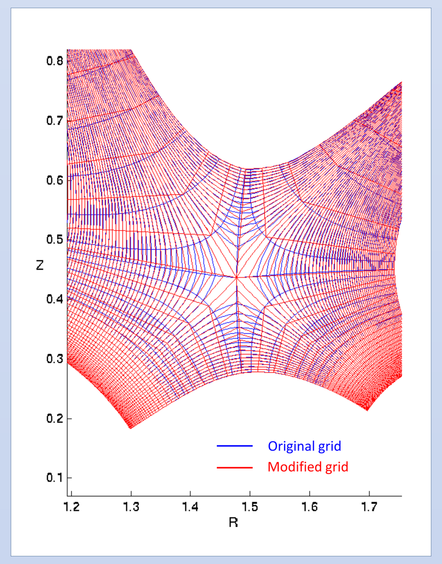
$$f(x) = \sum_p a_p x^p$$

- Solve least squares system for coefficients:

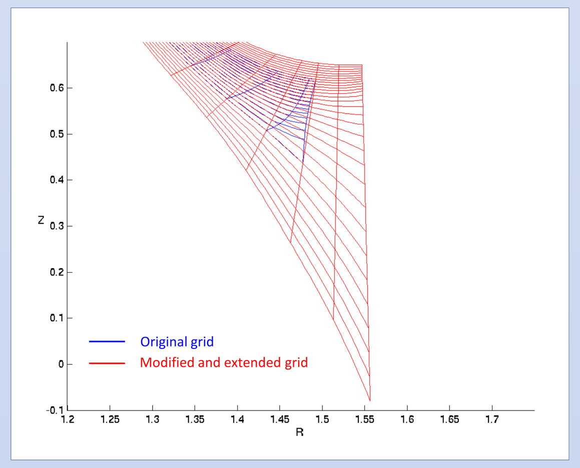
$$\int_V f(\xi) d\xi = \sum_p a_p \int_V x(\xi)^p d\xi$$

known for control volumes V centered at blue dots

computable from blue mapping

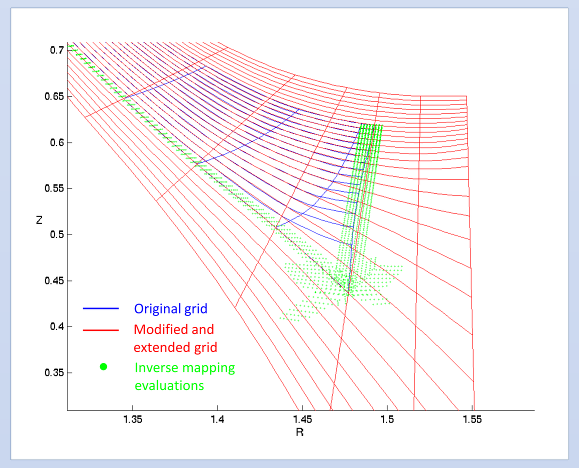


Grid modifications near the X point:
- Radial lines straightened
- Points within a prescribed distance from the X point smoothly redistributed



Extended left core block

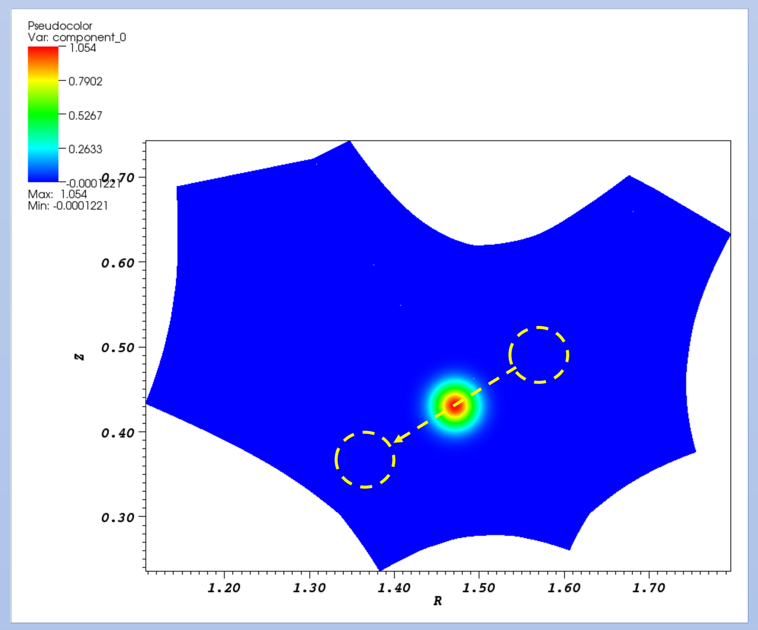
- Modified grid extended radially and poloidally
- Mapping of the computational to physical grid is defined by bicubic Hermite or spline interpolation



Inverse mapping evaluations

Mappings on the extended grids are inverted at the green points requested by neighboring blocks in the computation of the least squares interpolation coefficients

We are testing the accuracy of the discretization by propagating a Gaussian "blob" through the X point and performing grid convergence studies to estimate rates:



Poloidal cells across core (4h/2h) (2h/h)	Estimated conv. rate using bicubic Hermite interpolation	Estimated conv. rate using bicubic spline interpolation
512/1024 1024/2048	3.14 (L1) 3.62 (L2) 3.59 (Max)	3.56 (L1) 3.45 (L2) 2.79 (Max)
1024/2048 2048/4096	2.29 (L1) 3.00 (L2) 3.36 (Max)	3.77 (L1) 3.72 (L2) 3.77 (Max)
2048/4096 4096/8192	1.93 (L1) 1.71 (L2) 2.40 (Max)	2.77 (L1) 2.74 (L2) 3.22 (Max)

Fourth-order convergence is achieved for a blob advection test in a generic multiblock X point geometry with smooth mappings:

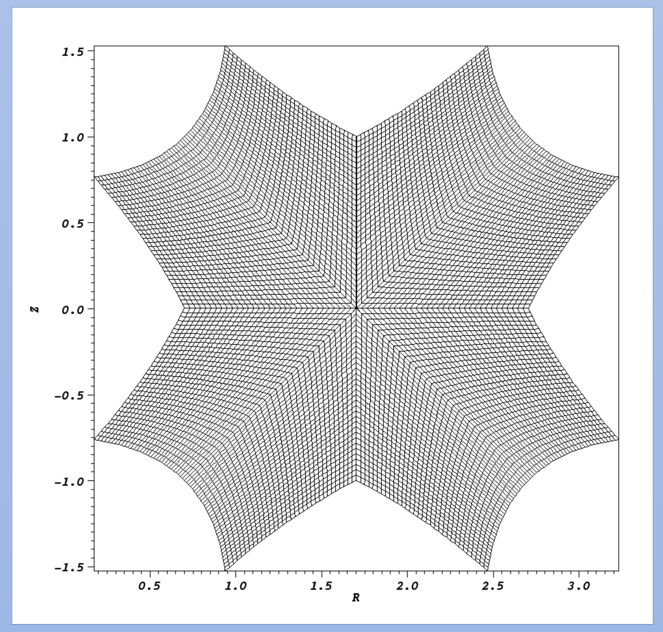
Test mapping:

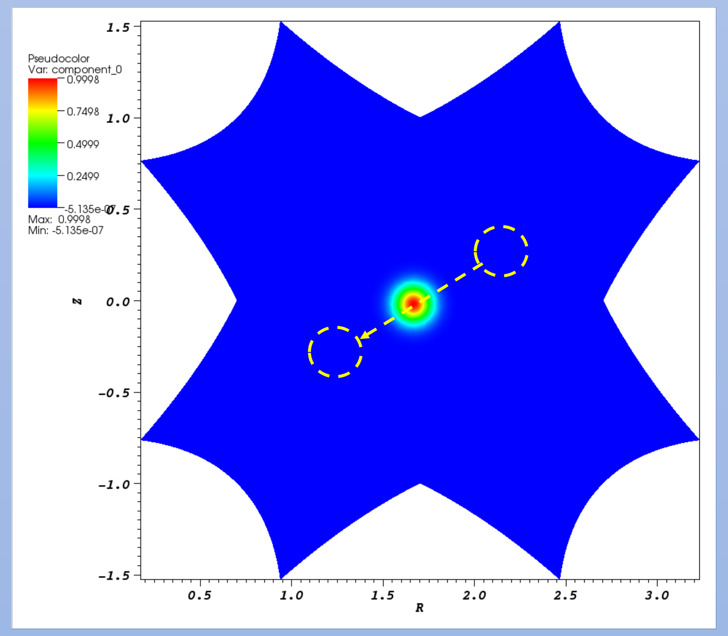
$$x_1' = \xi_1\left(3 + \left(1 - \alpha\xi_2\right)^2\right)/4$$

$$x_2' = \xi_2\left(3 + \left(1 - \alpha\xi_1\right)^2\right)/4$$

$$x_1 = ax_1' + bx_2'$$

$$x_2 = cx_1' + dx_2'$$





Grid block dimensions (4h/2h) (2h/h)	Estimated conv. rate
32 x 32 / 64 x 64 64 x 64 / 128 x 128	3.86 (L1) 3.40 (L2) 2.77 (Max)
64 x 64 / 128 x 128 128 x 128 / 256 x 256	4.09 (L1) 3.96 (L2) 3.82 (Max)
128 x 128 / 256 x 256 256 x 256 / 512 x 512	3.92 (L1) 4.01 (L2) 4.10 (Max)

LLNL-POST-503116