Sparse Line-Based Discontinuous Galerkin Discretizations and Efficient Time-Integration

Frontiers in Computation: New Methods for Complex Mechanics, Advanced Materials, Interfaces, and Stochastics

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Abstract

We present some recent developments in our work on high-order accurate methods for fluid and solid dynamics on unstructured meshes. While it is clear that DG and related methods are getting sufficiently mature to handle realistic problems, their computational cost is still at least a magnitude higher than low-order methods or high-order finite difference methods on similar grids. For some problems, explicit time-stepping or matrix-free implicit methods can be employed, but for many real-world problems and meshes full Jacobian matrices are required for the solvers to be efficient. Here, nodal-based Galerkin methods have a fundamental disadvantage in that they connect all unknowns inside an element, as well as all neighboring face nodes, even for first-order derivatives. This leads to a stencil size that scales like p^D for polynomial degrees p in D spatial dimensions. As a contrast, a standard finite difference method only involves nodes along neighboring lines, which gives a stencil size proportional to Dp, which can be magnitudes smaller ever for moderate values of p in 3-D.

In an effort to extend this stencil-size reduction to unstructured meshes, we propose applying the Galerkin formulation only for the 1-D problems that arise along each coordinate direction. We show how this can be done for fully unstructured meshes of hexahedra, and that the resulting scheme is drastically sparser than a corresponding DG scheme. For the second-order terms we use an LDG-inspired approach with consistent switches along all global lines, and we use modified iterative solvers that preserve most of the sparsity. In our numerical examples, we demonstrate optimal convergence for several problems including Poisson, convection, and the Euler equations. We show that super-convergence can be obtained if the systems are solved in split form, for an additional order of accuracy in the gradients. We study various time integrators for the compressible Navier-Stokes equations, in particular Implicit-Explicit Runge-Kutta solvers that reduce the size of the implicit problems, which we solve using a matrix-based Quasi-Newton method. This is appropriate in particular for time-accurate integration of LES-type problems, where a large part of the mesh is often uniform and a small number of boundary elements restrict the timestep for fully explicit solvers.