Designing A New Toolkit for Nonlinear Optimization

Tools and Theory for Nonlinearly Constrained Optimization

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Abstract

Nonlinearly constrained optimization is one of the most fundamental modeling paradigms in scientific computing. It not only models a range of important applications but also forms the basic building block of more complex optimization problems such as mixed-integer nonlinear optimization, and optimization with equilibrium constraints. Nonlinearly constrained optimization problems are conveniently formulated as

$$(P) \quad \underset{x}{\text{minimize}} \ f(x) \quad \text{subject to} \ \ c(x) = 0, \ x \geq 0.$$

We present a new toolkit for solving nonlinear optimization problems. The toolkit is written in `C++`, and designed to be a flexible and extensible open-source toolbox for nonlinearly constrained optimization. It is based on the robust trust-region methodology, and currently implements a variety of active-set methods and a range of globalization strategies in a modular fashion that allows algorithmic tuning to problem classes.

Modern Active-Set Trust-Region Methods. Trust-region methods solve a nonlinear optimization problem by iteratively solving a linear or quadratic model within a trust-region, adjusting the size of the region depending on the progress towards optimality. A typical trust-region subproblem around the current iterate x_k has the form

$$\underset{d}{\text{minimize}} \ q_k(d) \quad \text{subject to} \ \ c_k + \nabla c_k^T d = 0, \ x_k + d \geq 0, \ \|d\| \leq \Delta_k,$$

where $q_k(d)$ is a linear or quadratic approximation of the Lagrangian of (P), and $\Delta_k > 0$ is the trust-region radius. We implement a range of active-set algorithms, including sequential quadratic programming, and sequential linear/quadratic programming techniques. Active-set methods are particularly competitive for solving mixed-integer and degenerate optimization problems. We present a novel augmented Lagrangian approach that incorporates second-order multiplier updates.

Robust Globalization Strategies. To promote convergence from remote starting points, our toolkit implements a range of globalization strategies, including classical penalty function methods, modern non-monotone filter methods, and recently developed funnel ideas. Filter methods accept a step d whenever it reduces the objective function $f(x_k + d) - f(x_k) \leq -\sigma q_k(d)$, or the constraint violation $\|c(x_k + d)\| \leq \beta \|c(x_k)\|$, where $0 < \sigma < \beta < 1$. We indicate how these methods interact naturally with the trust-region methodology, and present some preliminary numerical results comparing their effectiveness.

Future Developments. We are working on new interior-point methods to complement our active-set implementations. In addition, we are developing feasibility restoration heuristics that provide fast progress toward the feasible region by taking first-order steps that reduce $\|c(x)\|$ without the need to solve computationally costly subproblems. Our long-term goal is the development of a broad range of parallel, robust nonlinear solvers.