Dimension reduction and measure transformation in stochastic simulations of coupled systems

Mathematical and Computational Tools for Predictive Simulation of Complex Coupled Systems Under Uncertainty

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Abstract

The modeling and simulation of complex coupled systems governed by multiple physical processes that may exist simultaneously across multiple scales and domains represent critical tools for addressing numerous science and engineering challenges. Models, by definition, are approximations of their target physical scenarios, and thus are prone to modeling errors; also, parametric uncertainties exist as a reflection of various limitations in experimental methods. The quantification of uncertainties thus constitutes a crucial requirement for predictive simulations of coupled systems to find useful applications in supporting scientific discovery and engineering.

We investigate in this work a computational framework for the efficient propagation of uncertainties through coupled models based on a general stochastic expansion methodology. We address the following two challenges that arise in extending stochastic expansion methods to coupled problems. The first challenge relates to the communication of information across physics, scale or domain interfaces. The process of condensing model predictions into physical parameters, solution patches, or other quantities to be communicated across interfaces often involves a compression of information, thus calling for probabilistic representations of exchanged information of reduced stochastic dimension. The second challenge is in the curse of dimensionality, in that the computational cost of expansion methods grows quickly with the stochastic dimension of the system, which can rapidly become large when uncertainties affect multiple model components.

We propose to systematically represent information by an adaptation of the Karhunen-Loeve decomposition as it is communicated across interfaces. Our adaptation permits the persistent seggragation of uncertainty between components of the system, thus allowing for an efficient albeit accurate solution in a reduced dimensional space. We propose to take advantage of this dimension reduction by carrying out algorithmic operations (including the construction of orthogonal polynomials and cubature rules) in terms of the reduced stochastic degrees of freedom defined at the interfaces, thus mitigating the growth in stochastic dimension as uncertainties are exchanged.

The poster will demonstrate the effectiveness of these techniques on a multiphysics problem relevant to nuclear reactors. This problem consists of a coupled system of two second-order elliptic equations, which describes the stationary transport of neutrons in a reactor with a temperature feedback. We accommodate uncertainties in both the neutron-transport and heat-transfer equations, and solve for the solution of the fully coupled problem by an iteration procedure that only involves the successive solution of the model components in their reduced stochastic dimension.