A fast direct solver for structured linear systems by recursive skeletonization

Applied Analysis and Computational Mathematics
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Abstract

We present a fast direct solver for structured linear systems based on multilevel matrix compression. Using the recently developed interpolative decomposition (ID) of a low-rank matrix in a recursive manner, we embed an approximation of the original matrix into a larger, but highly structured sparse one. The resulting compressed representation allows for efficient storage, fast matrix-vector multiplication, fast matrix factorization, and fast application of the inverse. The algorithm proceeds in two phases: a precomputation phase, consisting of matrix compression and factorization, followed by a solution phase to apply the matrix inverse. For boundary integral equations which are not too oscillatory, e.g., based on the Green's functions for the Laplace or low-frequency Helmholtz equations, both phases typically have complexity $O(N)$ in two dimensions, where N is the number of discretization points. In our current three-dimensional implementation, the corresponding costs are $O(N^{3/2})$ and $O(N \log N)$ for precomputation and solution, respectively. Numerical experiments show a speedup of ~ 100 for the solution phase over modern fast multipole methods; however, the cost of precomputation remains high. Thus, the solver is particularly suited to problems where large numbers of iterations would be required. Such is the case with ill-conditioned linear systems or when the same system is to be solved with multiple right-hand sides. Our algorithm is implemented in Fortran and being prepared for public release.