Advanced Optimization Techniques for Entropy-Based Moment Closures

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Abstract

Moment methods are a well-known solution technique for approximating kinetic equations because they simplify the kinetic description of the system and are naturally parallelizable. Entropy-based moment closures retain fundamental properties such as hyperbolicity, entropy dissipation, and positivity. However, they suffer from drawbacks such as difficulty in maintaining the realizability of numerical solutions and the computational burden of solving the defining optimization problem.

In the present work, we develop numerical algorithms for implementing entropy-based moment closures for linear kinetic equations for slab geometry. We present a numerical algorithm which preserves realizability recognizing that the associated optimization problem can only be solved approximately. Then we detail the difficulties in solving the defining optimization problem, describing the importance of the quadrature and illustrating the ill-conditioning of the problem near the realizable boundary. We propose an optimization algorithm with two main features to tackle these challenges. First, an adaptive quadrature is used to accurately evaluate the objective function and to update the kinetic scheme. Second, for moments near the realizable boundary, which test the limits of finite computing precision, a regularization technique is applied. We present simulation results which demonstrate the behavior of our algorithm.