Construction and properties of coarse discretization spaces by algebraic multigrid

"Constrained Energy Minimization Discretization and Multigrid Methods"

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Abstract

Multigrid (or MG) is becoming a method of choice to solve large sparse systems of algebraic equations that typically arise in the numerical solution of partial differential equations due to its potential for optimal cost. We first briefly review some necessary and sufficient conditions for an optimal MG iteration method. Next, we focus on the "algebraic" versions of MG (or AMG). This refers to the case when the hierarchy of (vector) spaces needed to build a MG is not given, hence it has to be constructed by the user, generally, in some problem dependent way. We present several approaches to construct AMG coarse spaces targeting finite element discretization matrices. In particular, we focus on element agglomeration AMG (or AMGe) as well as on aggregation AMG with smoothing (or SA AMG). Motivated by our theory (necessary conditions for optimal TG convergence), namely that the AMG constructed coarse spaces, corresponding to a good AMG solver, possess at least some weak approximation properties, our goal is to use the AMG-constructed coarse spaces also as upscaling (discretization) tool. We present extensions to the case of coarsening of pair of spaces needed in the mixed finite element method, by providing construction of coarse pair of Raviart–Thomas subspaces with guaranteed approximation properties. Both, the performance as well as the weak and strong approximation properties of the coarse spaces, corresponding to few of the described AMG methods are illustrated on examples of elliptic problems with high-contrast coefficients in standard and mixed formulation.