Efficient Solvers and Preconditioners for Linear Systems of Equations
Arising from Stochastic Galerkin Projections

Mathematical and Computational Tools for Predictive Simulation of Complex Coupled
Systems Under Uncertainty

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Abstract

Uncertainty quantification in any physical system is a challenging task. There are several
sources of uncertainty ranging from lack of knowledge about the system to inherent randomness
in the system. These physical systems are often modeled as partial differential equations (PDEs)
with random coefficients to represent uncertainty. Monte Carlo techniques are popular methods to
solve these problems as they only require solutions to the PDE for a given set of realizations of the
input random coefficients. However large samples of realizations are required to get good accuracy.

In the recent years, spectral stochastic finite element methods have become a popular alterna-
tive. These methods generate coupled linear systems of the form:

$$\sum_{i=0}^{N} \sum_{j=0}^{L} c_{ijk} K_j u_i = f_k, \quad \text{for } k = 0, \ldots, N, \quad (1)$$

where $N$ is the number of terms in the polynomial chaos expansion of the random response, $L$
is the number of terms in the polynomial chaos expansion of the uncertain input data, \{\(u_i\)\}_{i=0}^{N},
{\(f_k\)}_{k=0}^{N}, \{K_j\}_{j=0}^{L}$ are the coefficients of the polynomial chaos expansion of the solution, forcing and
stiffness matrix respectively and \(c_{ijk}\) are a set of coefficients dictated by the underlying random
variables. Since the coefficients $c_{ijk}$ vanish for certain combinations of the indices $i$, $j$ and $k$, the
system matrix derived from the above formulation has a particular block-sparsity structure, see
e.g. [1]. Because of this sparsity, Krylov subspace methods are a good choice to solve these systems
of linear equations. However, good preconditioners are required to solve these equations efficiently.

In this work, solution methods and preconditioners exploiting the block sparsity structure of
above equations based on Jacobi and Gauss-Seidel algorithms [2] are examined and compared with
traditional mean-based preconditioning. The linear systems are solved via multi-grid preconditioned
GMRES provided by the AztecOO and ML packages in Trilinos. The stochastic Galerkin systems
are formulated using the Trilinos package Stokhos and Albany application code developed at Sandia
National Labs.

References

[1] M. Pellissetti and R. Ghanem, Iterative solution of systems of linear equations arising in the

(2011).