Analytical and Experimental Results for Inexact Methods for Linear and Nonlinear Eigenvalue Problems

Krylov subspace and Schwarz methods for PDEs, control, and other problems

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Abstract

Two state-of-the-art methods for computing a single eigenvalue and eigenvector of a generalized algebraic eigenvalue problem $Av = \lambda Bv$ are Rayleigh quotient iteration (RQI) and the single-vector Jacobi-Davidson (JD) method. They exhibit local cubic convergence if the problem is Hermitian, and quadratic otherwise. In these methods, a sequence of linear systems needs to be solved. For example, in RQI, these linear systems are of the form $(A - \mu_k B)y = Bx_k$. The above-mentioned convergence rates are valid when these linear systems are solved exactly. Due to the size and structure of practical problems, these linear systems should be solved with a modern iterative method, namely some preconditioned Krylov subspace method. A natural question is: What tolerance needs to be used when approximating the solution of these linear systems, so that the same order of convergence of the exact methods (cubic or quadratic) can be maintained?

Other authors have shown that the answer to this question is that these tolerances be monotonically decreasing towards zero. In this talk we show that for commonly used Krylov subspace methods and an appropriate type of preconditioner, the tolerances need not go to zero. In fact, we show that these tolerances can be fixed, of some small magnitude, inversely proportional to the condition number of the eigenvector matrix of the coefficient matrix of the (inner) linear system in question. Thus, locally cubic and quadratic convergence can be achieved at a very reasonable computational cost. We present numerical experiments on a variety of problems illustrating our theoretical results.

We also present results on new understanding of the use of preconditioners and Krylov subspace methods for the inner solves of these inexact methods for generalized eigenvalue problems.

In addition, we report on some preliminary work addressing similar questions for the general nonlinear eigenvalue problem $T(\lambda)v = 0$.

(Joint work with Fei Xue).