Stable and efficient modeling of anelastic attenuation in seismic wave propagation[1]

Serpentine: Finite difference methods for wave propagation in second order formulation

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Abstract

Dissipative mechanisms in the earth lead to anelastic attenuation of seismic waves. This attenuation is commonly modeled by describing the earth as a viscoelastic solid, which has a quality factor (Q) that is constant over several decades in the frequency-domain. Such material behavior can be approximated in the time-domain by superimposing n standard linear solid (SLS) mechanisms.

Large computational resources are often required for including realistic viscoelasticity in three-dimensional seismic wave simulations. The reason is that the n-SLS viscoelastic model requires a number of so called memory variables to be evolved together with the primary dependent variables (velocities and stresses, or displacements). Each memory variable adds an extra differential equation into the system that governs seismic wave propagation, and the numbers of extra variables and equations are proportional to n. In the first order (velocity-stress) formulation of the viscoelastic wave equation, this approach leads to a system of $9 + 6n$ differential equations. To reduce the memory requirements, we instead base our numerical method on the second order (displacement) formulation. By observing that only the divergence of the viscoelastic stress tensor is needed, we derive an equivalent system of $3 + 3n$ differential equations. Hence, for three ($n = 3$) SLS mechanisms, the number of time-dependent differential equations at each grid point in space is reduced from 27 to 12.

We use the summation-by-parts principle to develop a finite difference approximation of the three-dimensional viscoelastic wave equation, subject to free surface or Dirichlet boundary conditions. Sufficient conditions on the material parameters in the viscoelastic model are derived to guarentee well-posedness of the viscoelastic wave equation. We prove that our scheme satisfies an energy estimate and therefore is stable under two conditions. First, the material parameters must satisfy the conditions for well-posedness and, secondly, the time step must satisfy a CFL-type time step restriction. The proof relies on the summation-by-parts property of the discretization, and also holds in the case of heterogeneous material properties.

Numerical experiments verify the accuracy and stability of the new scheme. Semi-analytical solutions for a homogeneous half-space problem and a layer over half-space problem (LOH.3) are used to demonstrate how the number of viscoelastic mechanisms and the grid resolution influence the accuracy. We find that three ($n = 3$) standard linear solid mechanisms usually are sufficient to make the modeling error smaller than the discretization error.

The new method has been implemented as part of version 2.1 of the open source software WPP, which also allows for grid refinements with hanging nodes as well as free surface boundaries on realistic topographies.

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