An Interior Decomposition Algorithm for Two-Stage Stochastic Convex Program, and Integration Formulae and Scenario Generation via Optimization

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Abstract

In the first part of the poster we will briefly present a decomposition algorithm for two stage stochastic convex programming problem that is developed in consideration of a heterogeneous and possibly unreliable parallel computing environment. The model framework is rather general as it only assumes the knowledge of a self-concordant barrier for the underlying convex feasible region defined by the constraints. For example, constraints defined by linear, quadratic, second order cone, semi-definite, or Chebyshev system are all a special case admitting this barrier.

In the second part of the poster we will present a novel technique based on setting up the problem of scenario generation in stochastic optimization as a semi-infinite linear programming problem. The technique naturally takes advantage of (if any) prior knowledge of the sparsity structure of the integrand with respect to its approximation by a polynomial, which is a major advantage of the proposed technique. The semi-infinite linear programming formulation framework is general enough to allow restrictions (e.g., assuming it is uniform over a polyhedral or ellipsoid) on the domain of the probability distribution while specifying a distribution. In the stochastic programming context, the scenarios are generated to approximate the expected value function of a stochastic program. We show that only k scenarios are needed to match k moments. An oracle-polynomial complexity of the scenario generation problem is proved. The oracle problem is a polynomial optimization problem. Consequently, to the best of our knowledge, for the first time we approach the problem of numerical integration using optimization. We present computational results showing that for a large number of test problems from the optimization and numerical integration literature the proposed technique generates better quality solutions than those possible by Monte-Carlo or Quasi-Monte-Carlo methods popular in stochastic optimization.