Slope Limiters for Finite Volume Schemes on Non-coordinate-aligned Meshes

Applied Analysis and Computational Mathematics

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Abstract

Many second-order accurate finite volume methods are based on the following 3 steps:

1. Construct a (limited) linear polynomial in every cell out of the given cell averages (u_{ij}^n).
2. Calculate the fluxes through the cell edges by solving a Riemann problem using states based on the linear polynomial.
3. Update the cell averages to (u_{ij}^{n+1}) using the fluxes through the edges.

In Step 1, a limiter is needed which adjusts the slope appropriately. The limiter prevents overshoots and oscillations that arise at discontinuities, as well as maintaining positivity, since for example if pressure becomes negative your code will crash. Limiters in one dimension are very well understood. In two dimensions for rectangular grids a one-dimensional limiter is usually applied in the x and y direction separately. This approach is very straightforward and usually works well. However, this splitting of x and y components of the gradient relies on the cell edges being aligned with the coordinate axes.

On non-coordinate-aligned grids a two-dimensional limiter is much more difficult. This is especially true for embedded boundary grids with 'cut cells', the motivation for this work. These cells vary considerably in terms of size and structure. The standard approach for these cells, or more generally for unstructured meshes, uses a scalar limiter, where the x and y components are decreased by a scalar until there are no overshoots (possibly the scalar is zero, resulting in a first order scheme). Our approach for limiting is based on the work by Berger, Aftosmis, and Murman [BAM05]. There, the authors develop the idea of formulating the limiter as an optimization problem: retain as much of the unlimited gradient as possible while fulfilling some monotonicity conditions in order to avoid overshoot. This allows us to use different scaling factors for the x and y components of the gradient in order to minimize diffusion.

We have worked on different formulations for the objective function and the constraints as well as on different ways to solve the resulting optimization problem. We currently formulate this as a linear programming problem and use the all-inequality Simplex method for its solution. So far we have results in two space dimensions on several test cases. On a smooth model problem with an exact solution the error in the cut cells using the LP limiter instead of using a scalar limiter is reduced significantly. We also show that the new limiter works well on flows with shocks. We are currently working on additional test problems as well as on extensions to three dimensions. We will also test our limiter on general unstructured grids.

References

[BAM05] M. Berger, M. J. Aftosmis, and S. M. Murman. Analysis of slope limiters on irregular grids. In *43rd AIAA Aerospace Sciences Meeting, Reno, NV*, 2005. Paper AIAA 2005-0490.