Fast Computation of Stochastic Transcendentals

Quantifying Prediction Fidelity in Multiscale Multiphysics Simulations

Kevin Long and Kaleb D. McKale
Department of Mathematics and Statistics

Texas Tech University
Lubbock, Texas

Abstract

The elementary operations of addition, subtraction, multiplication, and division involving truncated polynomial chaos expansions (PCE) are easily carried out through closed-form calculations on the PCE coefficients; square roots (defined appropriately) may be computed very efficiently by Newton's method. Computation of transcendental functions of PCE (hereafter called "stochastic transcendentals," or ST) is not such a simple matter. The usual methods of computing elementary transcendental functions on $\mathbb{R}$ through polynomial or rational approximations are usually not applicable for ST. Computation by nonintrusive spectral projection (NISP) in more than a few stochastic dimensions requires the expensive approximation of high-dimensional integrals by cubature. Debusschere *et al.* [3] have introduced a method by which ST are computed through numerical approximation to a line integral (LI); by reduction to an integration in one dimension, the LI method is considerably more efficient than NISP for large stochastic dimension.

We have introduced a new approach to computation of ST based on iterated means. Though new to the world of stochastic computation, the method of iterated means is a very old idea in mathematics, with important algorithms developed by Borchardt and Gauss in the 19th century and Archimedes in the 3rd century BCE. In the 1970s these algorithms were resurrected and improved by Carlson[2] and Brent[1] for use in extended-precision calculation of transcendentals on $\mathbb{R}$ and $\mathbb{C}$. All methods of this type are iterative, with each iteration requiring only the arithmetic operations and the square root.

Because the BG algorithm and its descendents use only the arithmetic operations and the square root (all of which are easily computable on PCE), and because the algorithm retains its accuracy over the entire real line, the potential for use in computation in a PCE setting is clear: one carries out the iterative procedure above not on a single real number, but on a PCE. We have conducted experiments showing excellent results for the arctangent and an algorithmic analysis showing that the BG algorithm has computational complexity superior to the line integration method.

References

[1] R. P. Brent. Fast multiple-precision evaluation of elementary functions. *Journal of the ACM*, 23, 1975.

[2] B. C. Carlson. An algorithm for computing logarithms and arctangents. *Mathematics of Computation*, 26, 1972.

[3] B. J. Debusschere, H. N. Najm, P. P. Pebay, O. M. Knio, R. G. Ghanem, and O. P. LeMaitre. Numerical challenges in the use of polynomial chaos representations for stochastic processes. *SIAM Journal on Scientific Computing*, 26, 2004.