M-adaptation

Mimetic Methods for Partial Differential Equations

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Abstract

The OASCR project "Mimetic Methods for PDEs", PI is M.Shashkov, is dedicated to development and analysis of numerical methods that incorporate fundamental mathematical and physical principles into numerical algorithms. The cornerstone of mimetic discretization methods is a discrete vector and tensor calculus that mimics crucial identities (e.g. $\operatorname{div}\operatorname{curl} = 0$) and theorems (e.g. Helmholtz decomposition theorems) of a continuum calculus. Mimetic methods are designed to work on polygonal and generalized polyhedral meshes. We describe briefly our research directions and focus in detail on one of them.

A unique feature of any MFD method on polyhedral meshes (including hexahedral meshes popular in engineering applications) is that it is a *rich parametric family* of discretization methods with equivalent properties such as stability, a size of the stencil, and a discretization order. This family often includes other methods as particular members, for instance, finite element (FE) methods. We will present a few tools for analysis of the family of MFD methods to find members with additional properties. A novelty that these tools bring to the field of discretization methods is the *method adaptation* (or the *m-adaptation*) through the choice of an optimal, in some sense, member of the MFD family.

We illustrate the m-adaptation with two applications. First, we consider modeling of acoustic and elastic wave propagation. The MFD method introduces from one parameter (acoustic equation and quadrilateral mesh) to 78 parameters (elastic equation and hexahedral mesh) for each mesh element. These parameters can differ from one element to another, which gives us a lot of flexibility for an optimization. The parameters can be chosen withing a large range of values without affecting the formal order of convergence of the MFD method. Therefore, we refer to them as *free parameters*. Two particular selections of the free parameters correspond to the conventional finite difference (FD) and FE methods. We show that a proper choice of the free parameters allows one to compensate for an error introduced by a time discretization to reduce significantly (ten times and more) a numerical dispersion or a numerical anisotropy, compared to the FD and FE methods.

Second, we consider an elliptic equation. The discrete maximum principle (DMP) is one of the most difficult properties to achieve in numerical methods, especially when the computational mesh is distorted to adapt and conform to the physical domain or the problem coefficients are highly heterogeneous and anisotropic. Violation of the DMP may lead to numerical instabilities such as oscillations, and to unphysical solutions such as heat flow from a cold material to a hot one. We derive sufficient conditions for the DMP and show how to find a member of the MFD family that satisfies these conditions. On simplicial meshes, we recover the conventional angle conditions. On general meshes, we describe theoretical and numerical approaches leading to optimization algorithms with small number of parameters.

This is the joint work with V.Gyrya, M.Manzini and D.Svyatskiy.