An Infeasible Constraint-Reduced Method for Quadratic Optimization

Interior-Point Algorithms for Optimization Problems with Many Constraints
(DOE Grant DESC0002218, PI: Dianne P. O'Leary)

Meiyun Y. He and André L. Tits
Department of Electrical and Computer Engineering
and Institute for Systems Research
University of Maryland, College Park, MD 20742

Abstract

Optimization problems with many more inequality constraints than variables arise in support-vector machine training (for data training), network problems, model predictive control, and finite discretizations of semi-infinite programming problems such as those arising from partial differential equation models, to name just a few application areas. In such problems, only a small percentage of the constraints are active at the solution, with all others being, in a sense, irrelevant. The idea behind *constraint reduction* in interior-point algorithms is to adaptively identify, at each iteration of the solution process, a small set of seemingly critical constraints, and use only those for computing the search direction. Because the computational cost is proportional to the number of such constraints, *if the working set can be appropriately chosen*, the resulting algorithm likely significantly outperforms the unreduced version in terms of solution time. Earlier work, in the context of linear optimization, showed promise.

The case of convex quadratic optimization (CQP) was first considered under a previous DOE ASCR grant: see [JOT10]. While the basic algorithm considered in [JOT10] requires the availability of a feasible initial point, an ℓ_1 penalty function is then used to circumvent this limitation. This penalty function is "exact", i.e., when the penalty parameter is sufficiently large, the penalized problem has the same solution as the original problem. However, no recipe is proposed in [JOT10] for determining an appropriate threshold value for this parameter.

In this talk, continuing along the lines of our work performed in the context of linear optimization during the first year of this project and presented at last year's ASCR meeting (see [HT11] for the journal version), we propose a scheme that adaptively assesses, as the iteration progresses, whether an increase of the penalty parameter is in order. We proved that whenever the primal and dual feasible sets have nonempty interiors, (i) after finitely many iterations, the penalty parameter stops being adjusted, and (ii) the sequence of iterates converges to an optimal (feasible) solution.

The power of the proposed algorithm is demonstrated on a model-predictive control application, where it is typical that no feasible intial point is available. Two main observations are:

- Our adaptive-penalty-enhanced constraint-reduction scheme yields a multifold speed-up, which in many applications (e.g., aerospace) may turn out to be the key to feasibility of the model-predictive control methodology. (Each successive optimization problem must be solved in a time that does not exceed the sample time.)
- In this application, the use of a *fixed* penalty parameter as was considered in [JOT10] is inadequate. Selecting a fixed "large" value, to make it likely that the (unknown) threshold is exceeded, results in a significantly increased computation time.

References

[JOT10] J.H. Jung, D.P. O'Leary, and A.L. Tits, Adaptive Constraint Reduction for Convex Quadratic Programming, *Computational Optimization and Applications*, 2010.

[HT11] M.Y. He and A.L. Tits, Infeasible constraint-reduced interior-point methods for linear optimization, *Optimization Methods and Software*, 2011.