Uncertainty Quantification for Inverse Ice Sheet Dynamics

Uncertainty Quantification for Large-Scale Ice Sheet Modeling
Large-Scale Optimization for Bayesian Inference in Complex Systems

Carsten Burstedde, Omar Ghattas, Tobin Isaac, Noemi Petra, Georg Stadler, Hongyu Zhu
Institute for Computational Engineering and Sciences
Departments of Geological Sciences and Mechanical Engineering
The University of Texas at Austin
Austin, TX 78701

Abstract

Modeling the dynamics of polar ice sheets is critical for projections of future sea level rise. Yet, there remain large uncertainties in the basal boundary conditions and in the non-Newtonian constitutive relations employed within ice sheet models. Here, we consider the problem of estimating uncertainty in the solution of (large-scale) ice sheet inverse problems within the framework of Bayesian inference. Computing the general solution of the inverse problem—i.e., the posterior probability density—is intractable with current methods on today's computers, due to the expense of solving the forward model (3D Stokes flow with nonlinear rheology) and the high dimensionality of the uncertain parameters (which are discretizations of the basal slipperiness field and the Glen's law exponent field). However, under the assumption of Gaussian noise and prior probability densities, and after linearizing the parameter-to-observable map, the posterior density becomes Gaussian, and can therefore be characterized by its mean and covariance.

The mean is given by the solution of a nonlinear least squares optimization problem, which is equivalent to a deterministic inverse problem with appropriate interpretation and weighting of the data misfit and regularization terms. To obtain this mean, we solve a deterministic ice sheet inverse problem; here, we infer parameters arising from discretizations of basal slipperiness and rheological exponent fields. For this purpose, we minimize a regularized misfit functional between observed and modeled surface flow velocities. The resulting least squares minimization problem is solved by an inexact Newton-CG method, which employs adjoints to compute the necessary gradients and Hessian actions.

The posterior covariance matrix is approximated by the inverse of the Hessian of the least squares cost functional of the deterministic inverse problem. Direct computation of the Hessian matrix is prohibitive, since it would require solution of as many forward Stokes problems as there are parameters. Therefore, we exploit the compact nature of the Hessian of the data misfit term, which has a spectrum that collapses to zero rapidly (stemming from the fact that the observations are informative about a low dimensional subspace of the parameter fields). This permits us to construct a low rank approximation of data misfit Hessian, which can be found at a cost (in number of Stokes solves) that does not depend on the problem size, thus providing scalability to problem sizes of practical interest.

We apply this method to quantify uncertainties in the inference of basal boundary condition and Glen's flow law parameters from synthetic observations of surface ice flow velocity.