Multigrid Preconditioners for Linear Systems Arising in PDE Constrained Optimization

Multilevel Techniques for Large-Scale Inverse Problems

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Abstract

This work focuses on the numerical solution of large-scale, distributed optimal control problems of the form

$$\begin{cases} \min \ \ \frac{1}{2}\|y - y_d\|^2 + \frac{\beta}{2}\|u\|^2 \ , \\ \text{subj. to} : e(y,u) = 0, \ u \in \mathcal{U}_{\text{ad}} = \{u \in \mathcal{U} \ : \ a \leq u \leq b\} \ . \end{cases} \tag{1}$$

with the equality-constraints $e(y,u) = 0$ representing a (system of) partial differential equation(s). The goal of this project is to develop efficient multigrid algorithms for the linear systems involved in the solution process of (1).

Multigrid methods have long been associated with large-scale linear systems, the paradigm being that the solution process can be significantly accelerated by using multiple resolutions of the same problem. However, the exact embodiment of the multigrid paradigm depends strongly on the class of problems considered, with multigrid methods for differential equations (elliptic, parabolic, flow problems) being significantly different from methods for problems like (1), where the linear systems often resemble integral equations.

In this talk we will present a number of model problems for which we were able to construct optimal order multigrid preconditioners (a concept that will be clarified), as well as problems for which we were not able to design preconditioners of optimal order quality. The test-problems include (a) linear and semi-linear elliptic constrained problems, (b) optimal control problems constrained by Stokes flow (both (a) and (b) without control-constraints), and (c) control-constrained problems with linear-elliptic PDE constraints.

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