Frequenstist Parameter Constraints Through Gaussian Processes: An Alternative to Monte Carlo Markov Chains

Machine Learning for Massive Scale Cosmology

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Abstract

Traditionally, data sets are translated into constraints on large parameter spaces using Monte Carlo Markov Chains. This method effectively integrates over the parameter space by performing a random walk in which steps are selected to seek out and remain in regions of high likelihood density. By exploring only regions of high likelihood density, Monte Carlo Markov Chains limit the number of times expensive likelihood functions need to be evaluated and save computing time and resources. While effective at yielding Bayesian constraints on parameter space, Monte Carlo Markov Chains make no guarantee that the parameter space will be thoroughly searched, nor do they allow for the possibility that the model used to translate between data and parameters is poorly chosen. If there is more than one region of high likelihood density, the random walk may find only one. If the data is poorly fit by the theory, the walk will still give a confidence interval consisting of the least bad regions of parameter space.

We attempt to address these shortcomings with an algorithm that uses Gaussian process methods to predict, based on a modest random sampling of parameter space, what points in parameter space will lie on the boundary of our desired confidence limit. This allows us to define our confidence limit in an absolute, frequentist sense, avoiding limits that are merely less awful than the points around them. Prediction by Gaussian processes also allows us to assign an uncertainty to predicted points in parameter space. Using this uncertainty, we can identify what regions of parameter space have been poorly explored and evaluate points in those regions, mitigating the danger of skipping over a hidden region of high likelihood density.

We test our algorithm's performance against Monte Carlo Markov chains using the example of the Wilkinson Microwave Anisotropy Probe's year 7 release of data mapping the anisotropy spectra of the cosmic microwave background. We test for the accuracy of the constraints found by both methods, the thoroughness of parameter space exploration, and the efficiency of the code in performing that exploration.