Extremal problems in combinatorics

Organizers: *Linyuan Lu* (lu@math.sc.edu), University of South Carolina
Yi Zhao (yzhao6@gsu.edu), Georgia State University

Abstract

In combinatorics, it is quite common to study extremal configurations which satisfy a certain property. The configurations could be graphs, hypergraphs, finite sets, etc. Extremality can be taken with respect to different parameters, such as order and size. For example, the Turán problem asks the maximum number of edges in a graph that forbids a given subgraph. More abstractly, we study how global properties of a configuration influence local substructures of the configuration.

Session 1:

- **Speaker:** *Aaron Dutle* (dutle@mailbox.sc.edu), University of South Carolina,

  Title: Realizations of Joint Degree Matrices

  Abstract: The Joint Degree Matrix of a graph is a matrix whose (i, j) entry is the number of edges connecting vertices of degree i to vertices of degree j. A given matrix is called realizable if it is the joint degree matrix of a simple graph. As in the case of degree sequences, there is a simple set of conditions for when a matrix is realizable, as well as an operation that can be used to transform any realization into any other realization. We discuss these problems, as well as the extremal question of when a joint degree matrix is uniquely realizable.

- **Speaker:** *Andrzej Dudek* (andrzej.dudek@wmich.edu), Western Michigan University,

  Title: On generalized Ramsey numbers of Erdős and Rogers

  Abstract: Extending the concept of Ramsey numbers, Erdős and Rogers introduced the following function. For given integers $2 \leq s < t$ let

  $$f_{s,t}(n) = \min\{\max\{|W| : W \subseteq V(G) \text{ and } G[W] \text{ contains no } K_s\}\},$$

  where the minimum is taken over all K_t-free graphs G of order n.

  In this talk, we show that for every $s \geq 3$ there exist constants $c_1 = c_1(s)$ and $c_2 = c_2(s)$ such that $f_{s,s+1}(n) \leq c_1(\log n)^{c_2}\sqrt{n}$. This result is best possible up to a polylogarithmic factor. We also show for all $t \geq s + 2 \geq 6$, there exists a constant c_3 such that $f_{s,t}(n) \leq c_3\sqrt{n}$. In doing so, we partially confirm an old conjecture of Erdős by showing that $\lim_{n\to\infty} \frac{f_{s+1,s+2}(n)}{f_{s,s+2}(n)} = \infty$ for any $s \geq 4$. This is joint work with John Retter and Vojta Rödl.

- **Speaker:** *Christian Avart* (cavart@gsu.edu), Georgia State University,

  Title: On the chromatic number of generalized shift graphs

  Abstract: Erdős and Hajnal introduced the shift graphs as graphs the vertices of which are the k-element subsets of $[n] = \{1, 2, \cdots, n\}$ (or of an infinite cardinal κ) and with two k-sets $A = \{a_1, a_2, \cdots, a_k\}$ and $B = \{b_1, b_2, \cdots, b_k\}$ joined if $a_1 < a_2 = b_1 < a_3 = b_2 \cdots a_k = n_{k-1} < b_k$. They determined the chromatic number of these graphs (called shift graphs). In this talk we consider the chromatic number of graphs defined similarly for other types of mutual position with respect to the underlying ordering. These are the results of a joint work with Thomasz Luczak and Vojtech Rödl.

- **Speaker:** *Gexin Yu* (gyu@wm.edu), The college of William & Mary,

  Title: Strong edge-colorings for k-degenerate graphs

Abstract: We prove that the strong chromatic index for each k-degenerate graph with maximum degree D is at most $(3k-1)D - k(k+1) + 1$. This confirms a conjecture made by Chang and Narayanan.

Session 2:

- **Speaker:** *Jie Han* (jhan22@student.gsu.edu), Georgia State University,

  Title: Minimum vertex degree threshold for loose Hamilton cycles in 3-graphs

  Abstract: A loose Hamilton cycle in 3-uniform hypergraphs (3-graphs) is a spanning cycle in which two consecutive edges share a single vertex. Recently Buss, Hán and Schacht proved that every 3-graph H on n vertices with the minimum co-degree at least $(7/16 + o(1))\binom{n}{2}$ contains a loose Hamilton cycle. We improve this result by giving an exact minimum degree threshold. This is a joint work with Yi Zhao.

- **Speaker:** *Yi Zhao* (yzhao6@gsu.edu), Georgia State University,

  Title: Minimum codegree threshold for Hamilton cycles in k-graphs

  Abstract: A Hamilton t-cycle in k-uniform hypergraphs (k-graphs) is a spanning cycle in which two consecutive edges share exactly t vertices. We determine the minimum $(k-1)$-degree in a k-graph that guarantees a Hamilton t-cycle whenever $k-2t$ divides $k-1$ and show it is best possible. This improves earlier asymptotic results of Kühn and Osthus, Hán and Schacht. This is a joint work with Jie Han.

- **Speaker:** *Travis Johnston* (johnstjt@email.sc.edu), University of South Carolina,

  Title: Turán Problems on Non-uniform Hypergraphs

  Abstract: A non-uniform hypergraph $H = (V, E)$ consists of a vertex set V and an edge set $E \subseteq 2^V$; the edges in E are not required to all have the same cardinality. The set of all cardinalities of edges in H is denoted by $R(H)$, the set of edge types. For a fixed hypergraph H, the Turán density $\pi(H)$ is defined to be $\lim_{n\to\infty} \max_{G_n} h_n(G_n)$, where the maximum is taken over all H-free hypergraphs G_n on n vertices satisfying $R(G_n) \subseteq R(H)$, and $h_n(G_n)$, the so called Lubell function, is the expected number of edges in G_n hit by a random full chain. This concept, which generalizes the Turán density of k-uniform hypergraphs, is motivated by recent work on extremal poset problems.

  Several properties of Turán density, such as supersaturation, blow-up, and suspension, are generalized from uniform hypergraphs to non-uniform hypergraphs. Other questions such as "Which hypergraphs are degenerate?" are more complicated and don't appear to generalize well. In addition, we completely determine the Turán densities of $\{1, 2\}$-hypergraphs.

- **Speaker:** *Eva Czabarka* (czabarka@math.sc.edu), University of South Carolina,

  Title: Large families for the diamond problem based on Abelian groups

  Abstract: There is much recent interest in excluded subposets. Given a fixed poset P, how many subsets of $[n]$ can found without a copy of P realized by the subset relation? The hardest and most intensely investigated problem of this kind is when P is a *diamond*, i.e. the power set of a 2 element set. In this paper, we show infinitely many asymptotically tight constructions using random set families defined from posets based on Abelian groups. They are provided by the convergence of Markov chains on groups. Such constructions suggest that the diamond problem is hard.

Session 3:

- **Speaker:** *Laszlo Szekely* (laszlo@mailbox.sc.edu), University of South Carolina,

  Title: Threshold functions for distinct parts: revisiting Erdős–Lehner

  Abstract: We study four problems: put n distinguishable/non-distinguishable balls into k non-empty distinguishable/non-distinguishable boxes randomly. What is the threshold function $k = k(n)$ to make almost sure that no two boxes contain the same number of balls? The non-distinguishable ball problems are essentially equivalent to the Erdős–Lehner asymptotic formula for the number of partitions of the integer n into k parts with $k = o(n^{1/3})$. The problem is motivated by the statistics of an experiment, where we only can tell whether outcomes are identical or different. This is joint work with Éva Czabarka and Matteo Marsili,

- **Speaker:** *Richard Anstee* (anstee@math.ubc.ca), University of British Columbia,

  Title: Critical Substructures of Configurations

  Abstract: We consider the problem of forbidden configurations. Define a matrix to be *simple* if it is a (0,1)-matrix with no repeated columns. Such a matrix can be interpreted as the element-set incidence matrix of a family of subsets of $\{1, 2, 3, \ldots, m\}$. For a given (0,1)-matrix F, we say a matrix A has no *configuration* F if there is no submatrix of A which is a row and column permutation of F. Given m and a configuration F, we seek a bound $\text{forb}(m, F)$ on the number of columns in an m-rowed simple matrix which has no configuration F.

  In this talk we consider how certain critical substructures can determine the bounds. Joint with S.N. Karp, C.G.W. Meehan, M. Raggi.

- **Speaker:** *Linyuan Lu* (lu@math.sc.edu), Universithy of South Carolina,

  Title: Unrolling residues to avoid progressions

  Abstract: Van der Waerden theorem states that for any r and k there is an N so that if $n \geq N$ then for any coloring of $[n] = \{1, 2, \ldots, n\}$ using r colors there must be an arithmetic progression of length k (or k-APs for short) which is monochromatic. In this talk, we consider the problem to minimize the number of monochromatic k-APs among all r-colorings of $[n]$. We show how to extend colorings of $\mathbb{Z}/m\mathbb{Z}$ which avoid nontrivial k-APs to colorings of $[n]$ by an unrolling process. In particular, by using residues to color $\mathbb{Z}_{m\mathbb{Z}}$ we produce the best known colorings for minimizing the number of monochromatic k-APs for coloring with r colors for several small values of r and k. This a joint work with Steve Bulter, Ron Graham, and Xing Peng.