# Topics in Analysis

- **Title**: The asymptotic behavior of Rayleigh quotients in variable exponent spaces and applications
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**Abstract:** The continuity of the Luxemburg norm of the gradient in $L^{p(\cdot)}(\Omega)$ with respect to $p(\cdot)$ is discussed via De Giorgi's $\Gamma$-convergence. As a consequence, the minimum values of closely related "Rayleigh quotient" type functionals are shown to converge to a certain quantity associated to the $\Gamma$-limit. Further, the asymptotic behavior of Rayleigh quotients involving both Luxemburg norms and modulars is studied as $p(\cdot) \to \infty$. In a particular case, we recover a well-known result of Juutinen, Lindqvist and Manfredi regarding the principal frequency of $\Delta_\infty u := \sum_{i,j=1}^{N} u_{x_i} u_{x_j} u_{x_i x_j}$.

- **Title**: An Analysis of a Porous Medium Equation
**Speaker:** *Koffi B. Fadimba* (KoffiF@usca.edu), University of South Carolina Aiken

**Abstract:** The equation

$$\phi \frac{\partial S}{\partial t} - \nabla \cdot (k(S) \nabla S) = Q(S) \tag{1}$$

is a more general form of the classical Porous Medium Equation (PME)

$$\frac{\partial S}{\partial t} - \Delta \left(S^m\right) = Q(S) \tag{2}$$

Equation (1) is obtained through a mathematical modeling of an immiscible and incompressible two-phase flow through a porous medium, with $S$ the saturation of the invading fluid. In the model described by equation (1), a simplification, among others, is that the the porosity of the medium, $\phi$, is independent of time, or changes very little with time ($\phi_t \approx 0$).

In this work, we consider the case where the porosity $\phi = \phi(x, t)$ is a function of both the spatial variable $x$ and the temporal variable $t$. Then the equation is of the form

$$\frac{\partial (\phi S)}{\partial t} - \nabla \cdot (k(S) \nabla S) = Q(S). \tag{3}$$

We perform a mathematical analysis of equation 3, based on the assumption that the diffusion coefficient satisfy $k(0) = k(1) = 0$. We then apply our analysis to the classical saturation equation.

- **Title**: Enrichment of NURBS Basis by the NURBS Geometrical Mapping that Generates Singular Functions for Isogeometric Analysis
**Speaker:** *Hyunju Kim* (hkim22@uncc.edu), University of North Carolina
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**Abstract:** In order to handle the singularities arising in the PDEs, it was introduced to a novel NURBS geometrical mapping that generates singular functions. In this presentation, we consider how to use the proposed mapping method in IGA of elliptic problems and elasticity containing singularities without changing the design mapping. For this end, we embed the mapping method into the standard IGA that uses NURBS basis functions for improved computational solution.