Fault Tolerant Algorithms for Heat Transfer Problems

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Motivations

- Why fault tolerance?

  1. The new generation of Exascale HPC Systems have increasingly complex architectures
  2. The Mean Time Between Failures (MTBF) of previous parallel computers used to be several weeks and now has dramatically decreased to just few minutes (Engelmann and Al Geist, 2002)
  3. The Message Passing Interface (MPI) does not address the FT issue which is NOT an option for long-running and critical Applications

The Parabolic 3D HEAT equation I

- The 3D heat equation writes

$$\frac{\partial u}{\partial t} = \Delta u + F(x,y,z,t),\ (x,y,z,t) \in \Omega \times (0,T)$$
$$u_{|\partial\Omega} = g(x,y,z),\ u(x,y,z,0) = u_o(x,y,z).$$

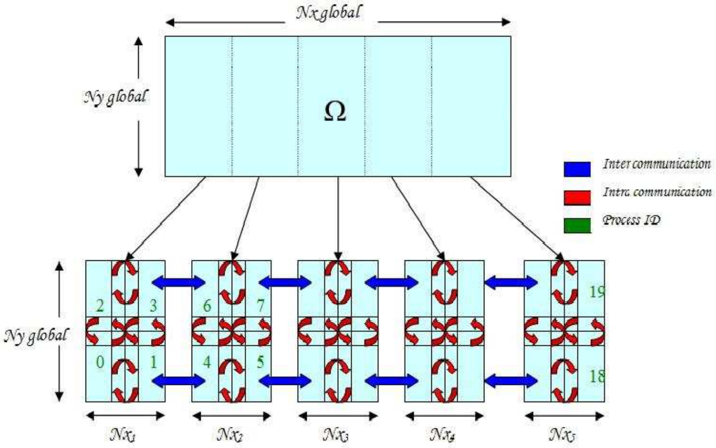
We suppose that the time integration is done by a first order implicit Euler scheme,

$$\frac{U^{n+1} - U^n}{dt} = \Delta U^{n+1} + F(x,y,z,t^{n+1}), \tag{1}$$

and that Ω is partitioned into N subdomains $\Omega_j,\ j = 1..N$.

The Parabolic 3D HEAT equation II

- Example : Domain Decomposition with 5 subdomains allocated to 5 distributed computing nodes



The Parabolic 3D HEAT equation III

- Why heat transfer problem?

1. Long time running numerical simulations
2. Reversibility in time not so easy (numerical instability)
3. ill-posed by nature
4. Reconstruction process of the lost data quite complex and challenging
5. NOT the case for
   $\not\Leftrightarrow$ Elliptic operators
   $\not\Leftrightarrow$ Linear Hyperbolic operators

Our Fault Tolerant Approach I

- The design of Fault Tolerant applications is based on 2 complementary features:

  1. Guaranty that if one or several machines get disconnected the code can still be executed
     $\Longrightarrow$ (A) Middleware Fault Tolerant-MPI (FT-MPI) (Fagg et al, International J. of High Performance Computing Applications, 2005)

  2. Provide an algorithm that can restart the time integration from the data that are available in the distributed memory or file system
     $\Longrightarrow$ (B) Non-Blocking, Periodic and Uncoordinated checkpointing

Our Fault Tolerant Approach II

- (A) FT-MPI:
  1. Ensures the Application will keep on running in presence of failures by respawning the failed processes. It does not provide any Checkpointing features.
  2. Responsibility of the Application to recover the data-structures on the crashed processes.
  3. The user has the ability to checkpoint only the minimum required data to restart the whole Application.
- (B) Non-Blocking / Periodic / Uncoordinated Checkpointing:
  1. Implementation at Application-Level
  2. Two groups of processes:
     $\Longrightarrow$ the *solver* group which actually solves the problem and perform data checkpointing
     $\Longrightarrow$ the *spare* group which, so far, only saves the checkpoints coming from the *solver* processes

Numerical Reconstruction Algorithms I

- Now, we assume that we have detected a failure and need to restart the computation of the time dependent problem.
- From the data stored in the spare processors main memory, we can rebuild in parallel the lost data on a subdomain using the following techniques:
  1. The Forward Implicit Time Integration
  2. The Backward Explicit Time Integration

Numerical Reconstruction Algorithms II

- The Forward Implicit Time Integration:

  $\Longrightarrow$ Save U at each K time step interval and additionnaly, BC at each time step

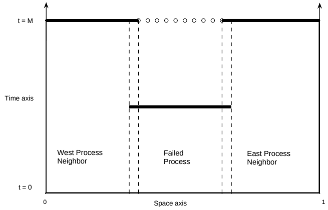


Figure: Available data on main memory processors before starting the reconstruction algorithms.

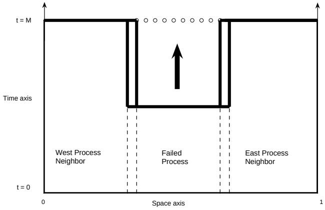


Figure: Reconstruction procedure in one dimension using forward time integration.

Numerical Reconstruction Algorithms III

- The Backward Explicit Time Integration:

$\Longrightarrow$ Save U only at each K time step interval

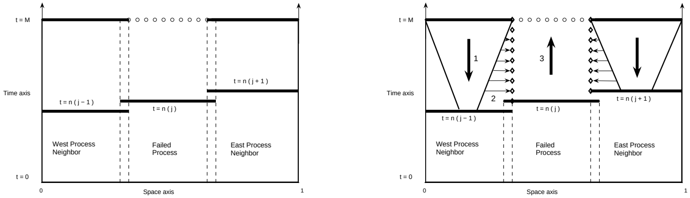


Figure: Available data on main memory processors before starting the reconstruction algorithms.

Figure: Reconstruction procedure in one dimension using explicit backward time stepping.

- Reconstruction Procedure in 3 successive computational steps

Numerical Reconstruction Algorithms IV

1. Explicit Backward Integration

$$U^n \; = \; U^{n+1} \; - \; dt \; \Delta U^{n+1} \; - F(x,y,z,t^{n+1}). \qquad (2)$$

   - The solution is granted by the Implicit Forward Integration in time (1)
   - This works only for few time steps!
   - Continue then with an hyperbolic regularization, the Telegraph equation (Eckhaus and Garbey, SIAM, 1990)

$$\epsilon \frac{\partial^2 U}{\partial t^2} - (\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}) + \frac{\partial U}{\partial t} = F(x,y,z,t), x\ y\ z \in (0,1)^3, t \in (0,T) \qquad (3)$$

   - The smaller ϵ, the more unstable the scheme (3) and the flatter the cone of dependence
   - The smaller ϵ, the better the asymptotic approximation
2. Space Marching Scheme (Beck and Murio, SIAM, 1994) on each direction: standard procedure in inverse heat problem

$$\frac{U^n_{j+1} \; - \; 2\; U^n_j \; + \; U^n_{j-1}}{h^2} = \frac{U^{n+1}_j - U^{n-1}_j}{2\; dt} + F^n_j, \qquad (4)$$

3. Application of the Implicit Forward Integration reduced to the failed subdomain

Numerical Reconstruction Algorithms V

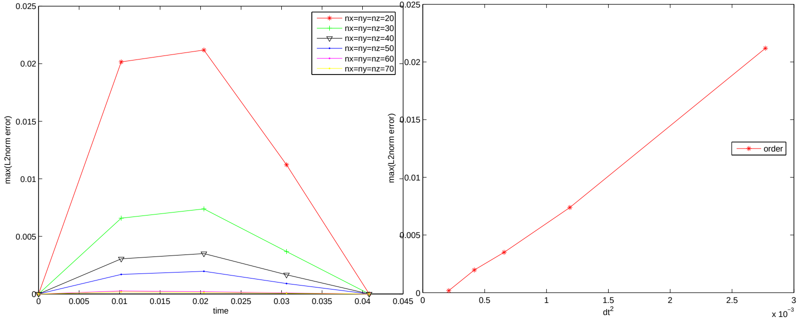


Figure: Numerical accuracy and order of the 3D overall reconstruction algorithm.

Performance Comparisons I

- Performance of the 2 PFT Algorithm on a cluster of 56 dual Itanium2 processor (900MHz) nodes, all connected to a gigabit ethernet network.

  1. *small*$\Longleftrightarrow(10 \times 10 \times 50)$
  2. *medium*$\Longleftrightarrow (15 \times 15 \times 76)$
  3. *large*$\Longleftrightarrow (18 \times 18 \times 98)$

Performance Comparisons II

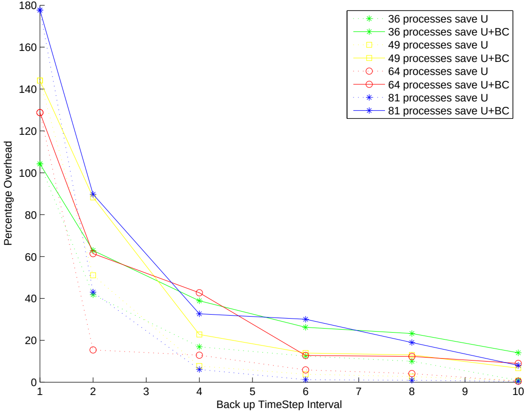


Figure: Overhead of saving the local solutions with 36, 49, 64 and 81 processes: **Small** data set.

Performance Comparisons III

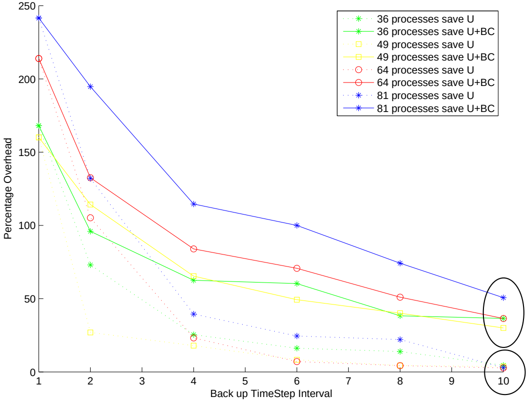


Figure: Overhead of saving the local solutions with 36, 49, 64 and 81 processes: **Medium** data set.

Performance Comparisons IV

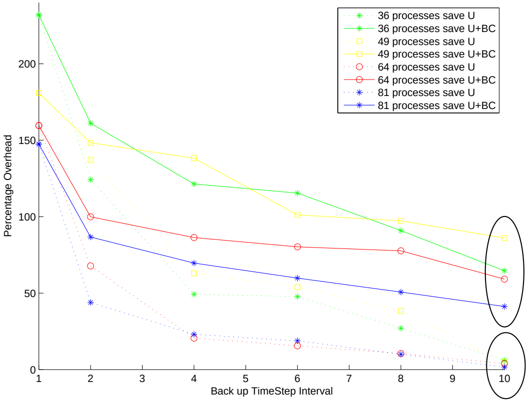
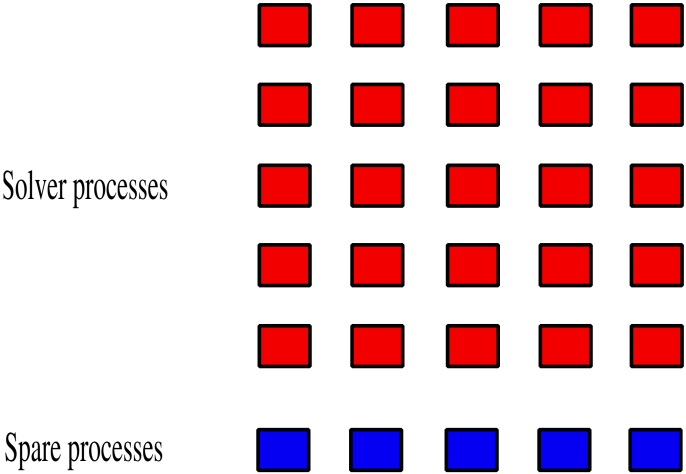


Figure: Overhead of saving the local solutions with 36, 49, 64 and 81 processes: **Large** data set.

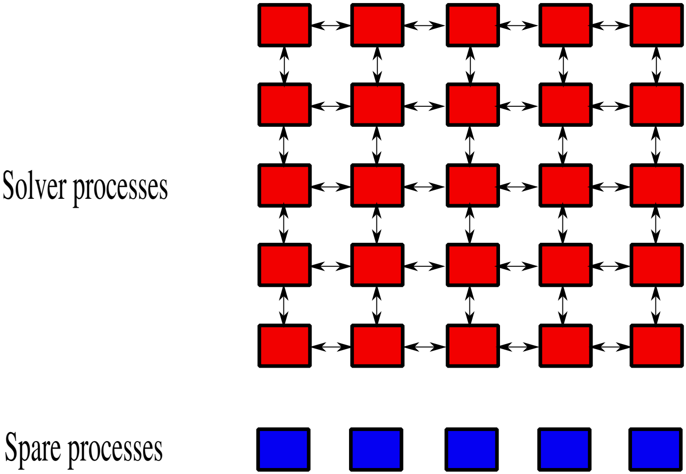
Demo I

Time Step = 1 ; Checkpoint Interval = 10



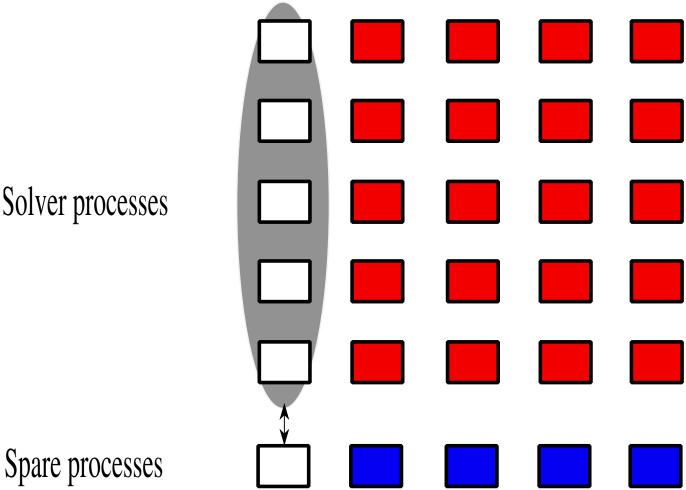
Demo II

Time Step = 1 ; Checkpoint Interval = 10



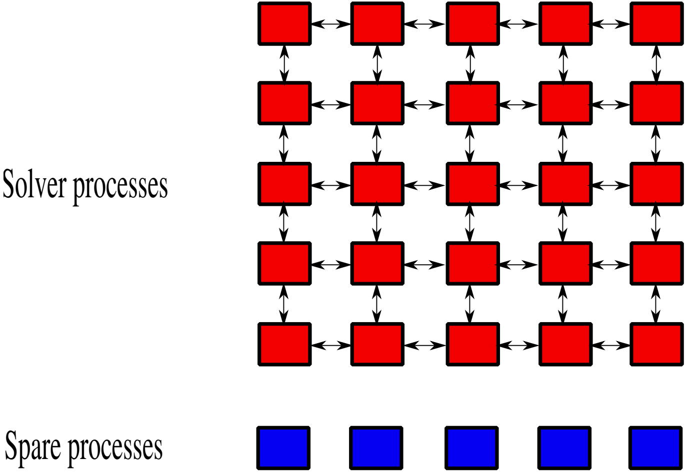
Demo III

Time Step = 1 ; Checkpoint Interval = 10



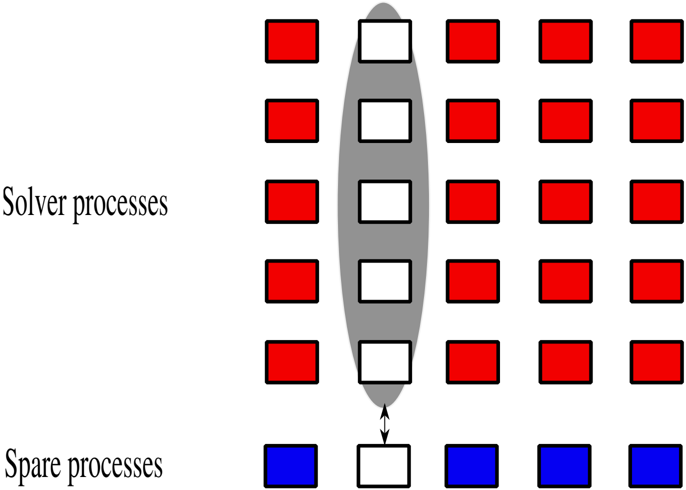
Demo IV

Time Step = 2 ; Checkpoint Interval = 10



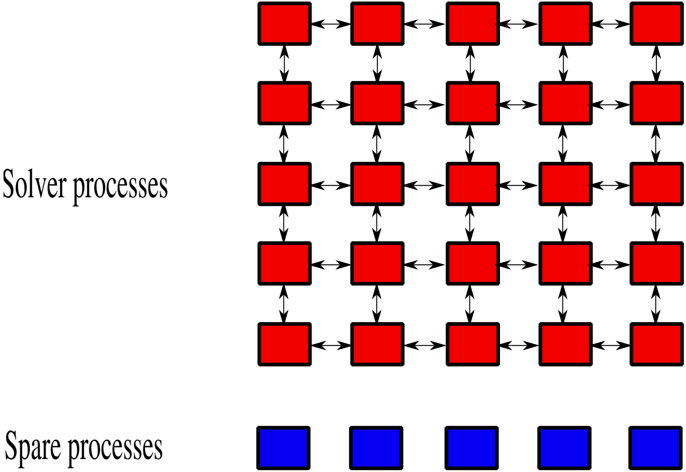
Demo V

Time Step = 2 ; Checkpoint Interval = 10



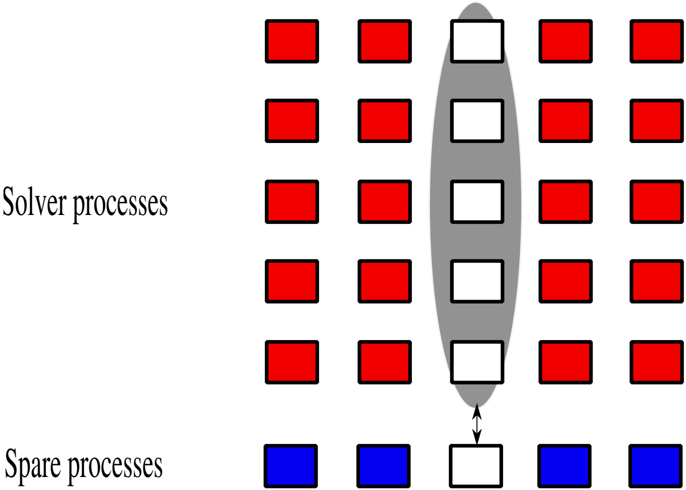
Demo VI

Time Step = 3 ; Checkpoint Interval = 10



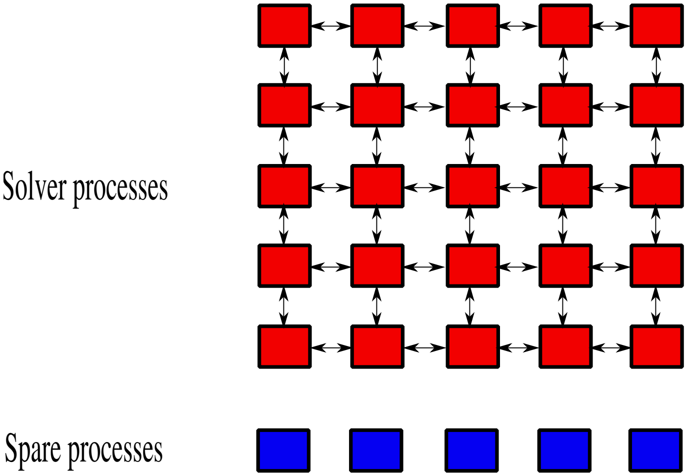
Demo VII

Time Step = 3 ; Checkpoint Interval = 10



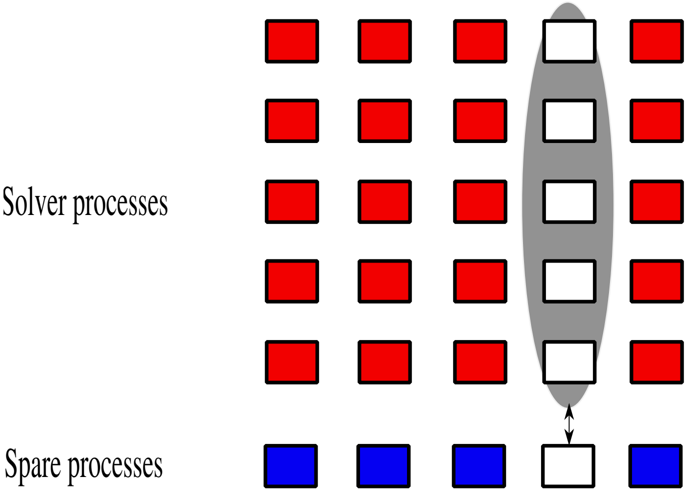
Demo VIII

Time Step = 4 ; Checkpoint Interval = 10



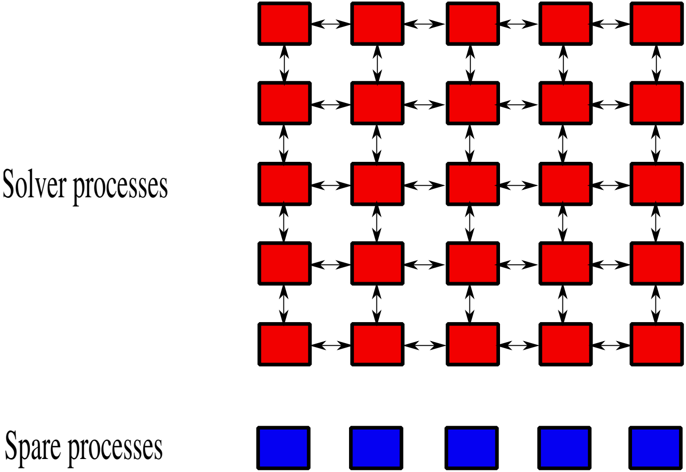
Demo IX

Time Step = 4 ; Checkpoint Interval = 10



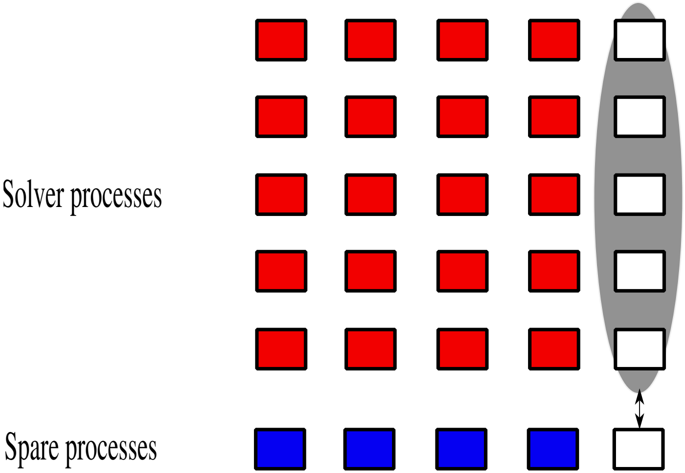
Demo X

Time Step = 5 ; Checkpoint Interval = 10



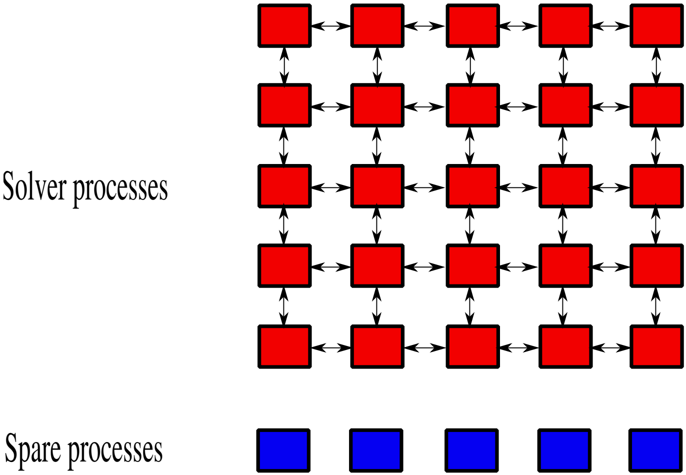
Demo XI

Time Step = 5 ; Checkpoint Interval = 10



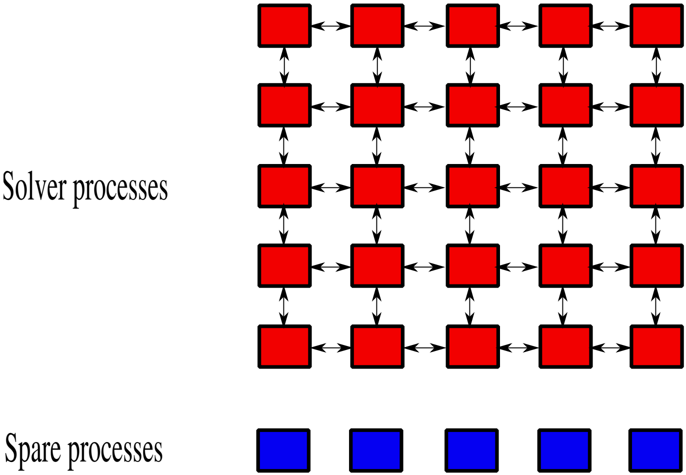
Demo XII

Time Step = 6 ; Checkpoint Interval = 10



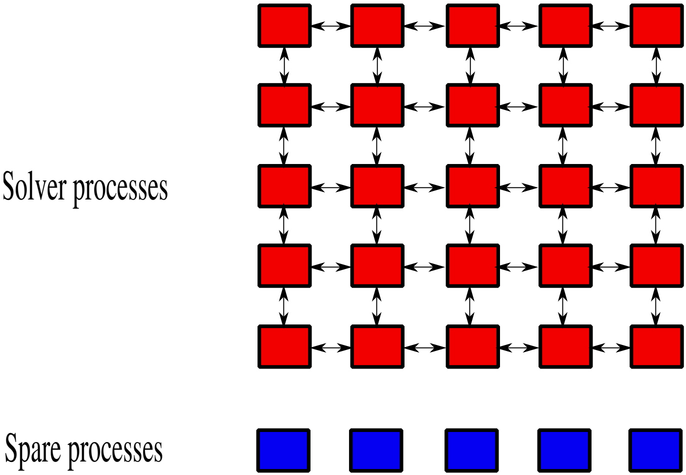
Demo XIII

Time Step = 7 ; Checkpoint Interval = 10



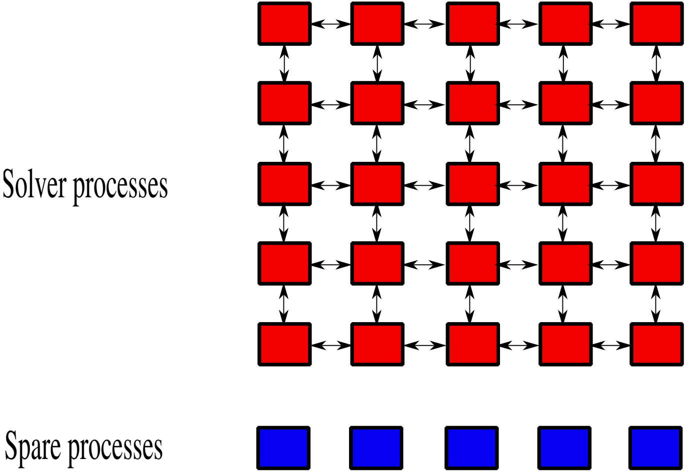
Demo XIV

Time Step = 8 ; Checkpoint Interval = 10



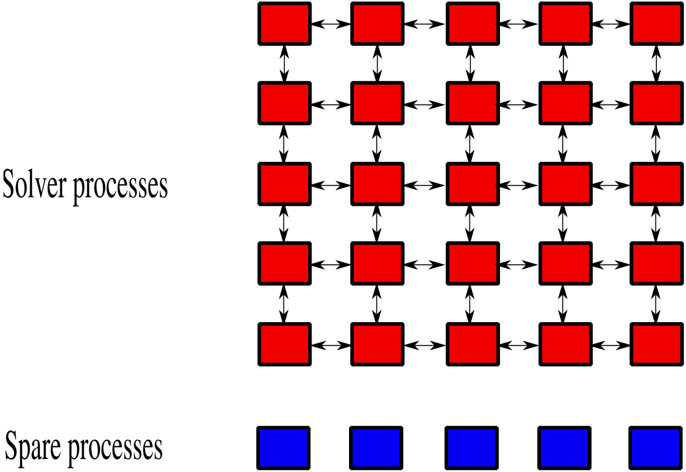
Demo XV

Time Step = 9 ; Checkpoint Interval = 10



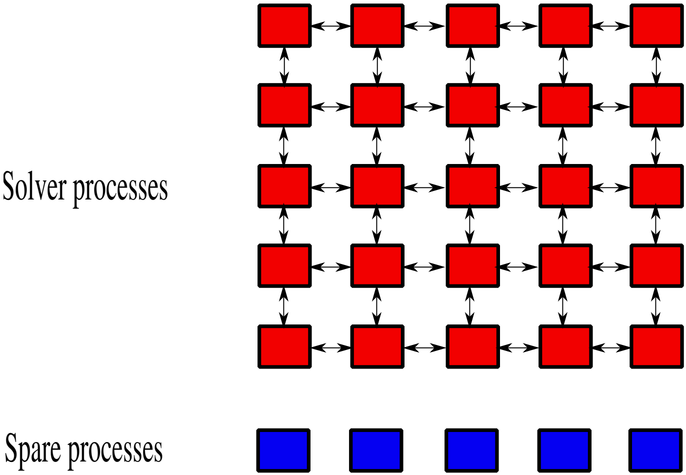
Demo XVI

Time Step = 10 ; Checkpoint Interval = 10



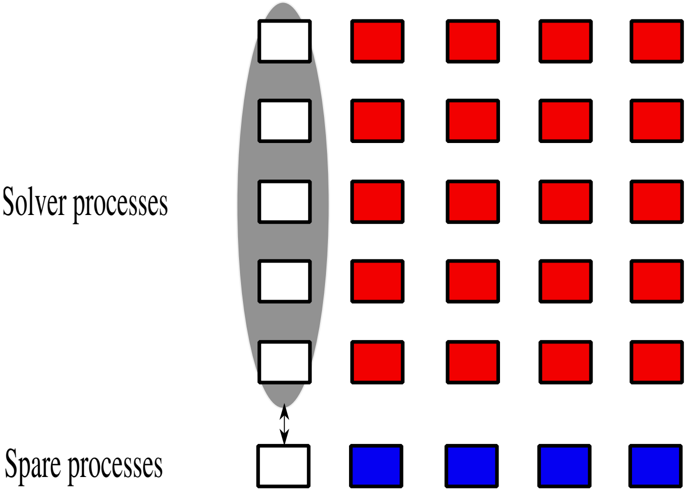
Demo XVII

Time Step = 11 ; Checkpoint Interval = 10



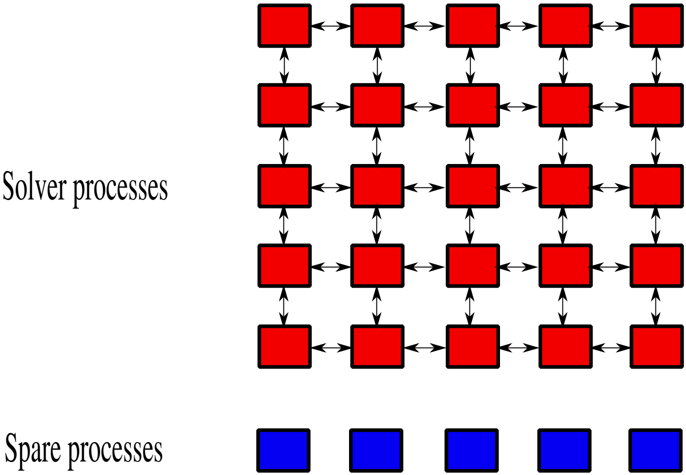
Demo XVIII

Time Step = 11 ; Checkpoint Interval = 10



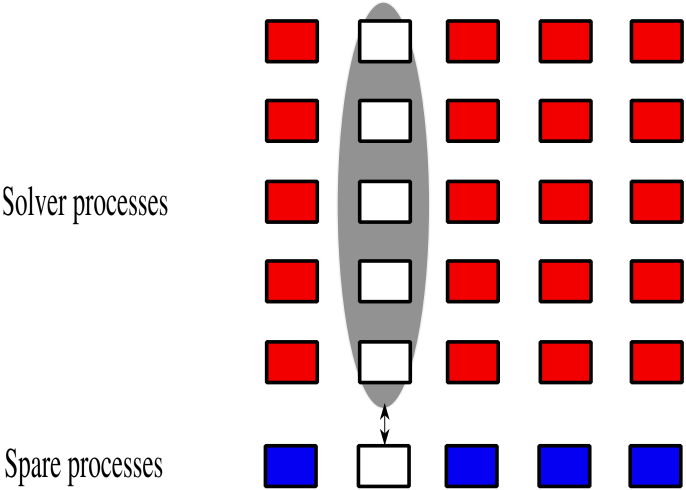
Demo XIX

Time Step = 12 ; Checkpoint Interval = 10



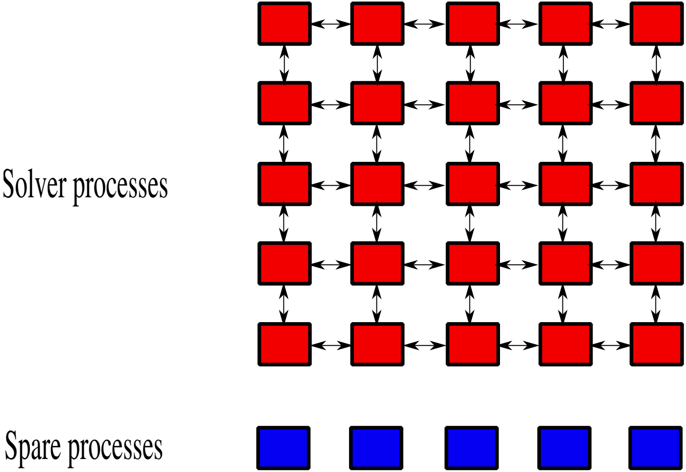
Demo XX

Time Step = 12 ; Checkpoint Interval = 10



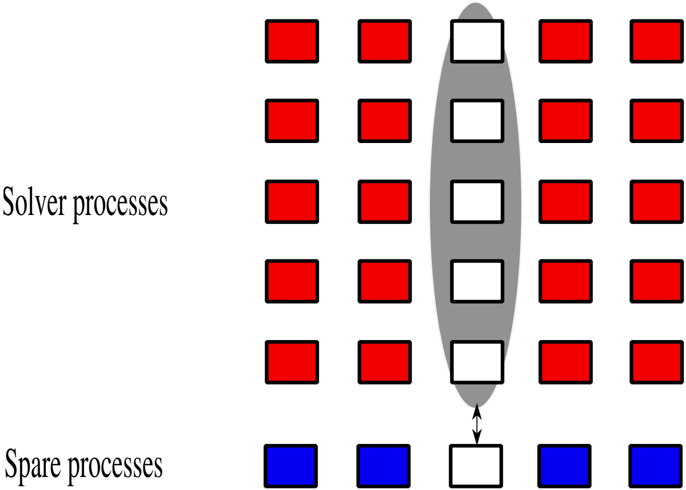
Demo XXI

Time Step = 13 ; Checkpoint Interval = 10



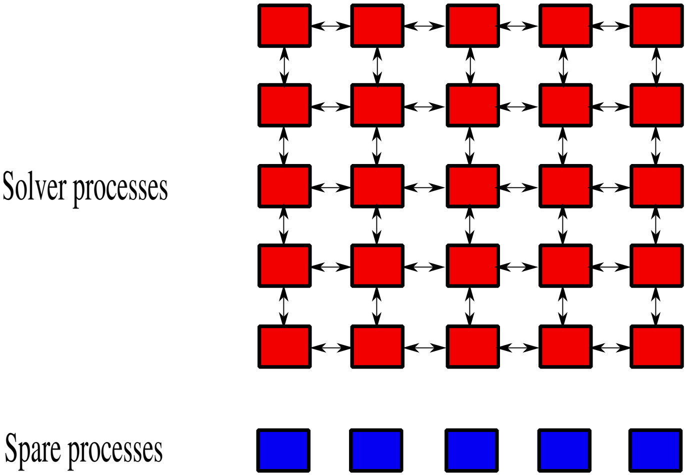
Demo XXII

Time Step = 13 ; Checkpoint Interval = 10



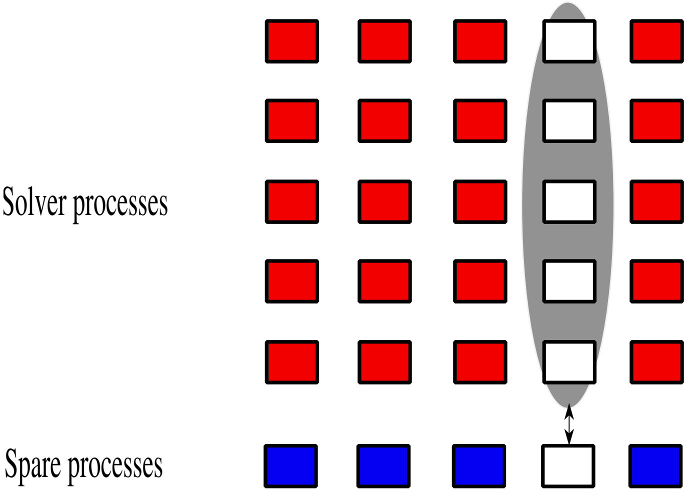
Demo XXIII

Time Step = 14 ; Checkpoint Interval = 10



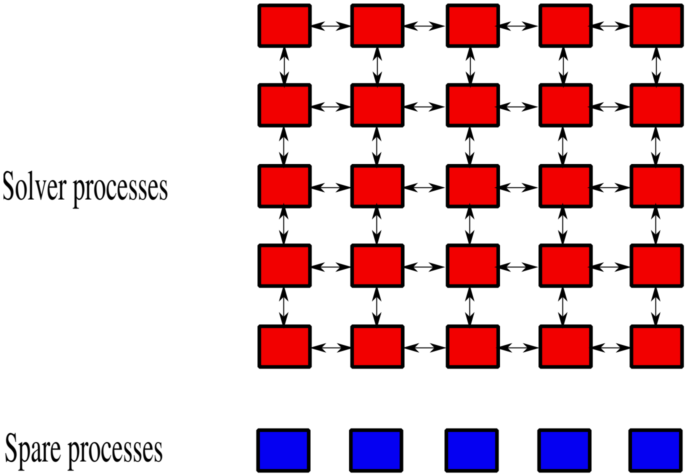
Demo XXIV

Time Step = 14 ; Checkpoint Interval = 10



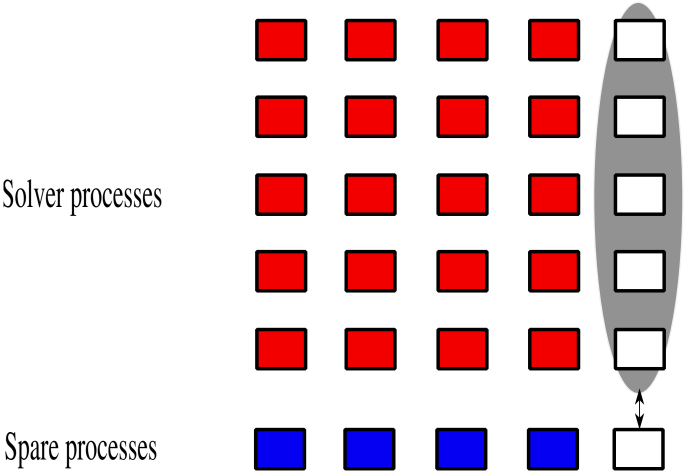
Demo XXV

Time Step = 15 ; Checkpoint Interval = 10



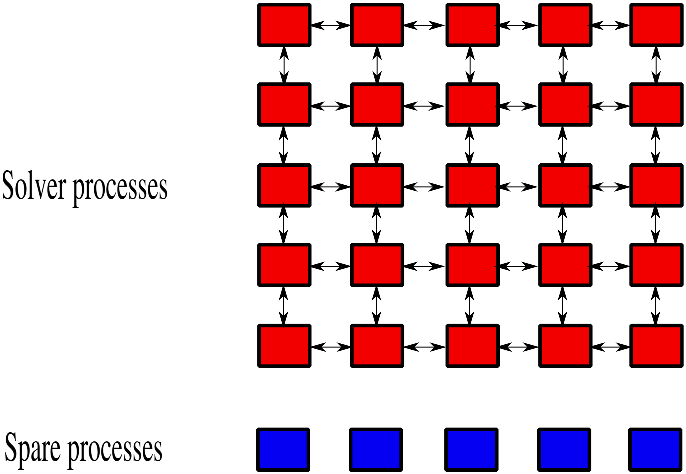
Demo XXVI

Time Step = 15 ; Checkpoint Interval = 10



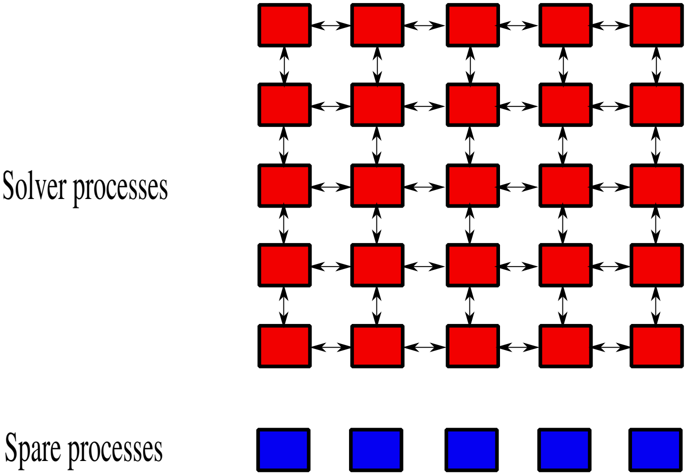
Demo XXVII

Time Step = 16 ; Checkpoint Interval = 10



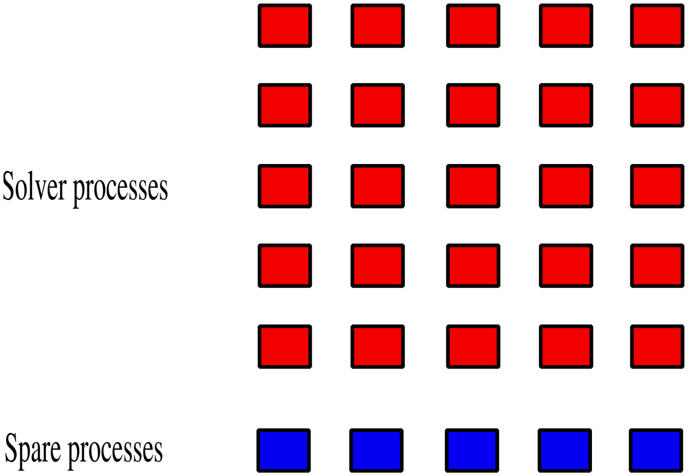
Demo XXVIII

Time Step = 17 ; Checkpoint Interval = 10



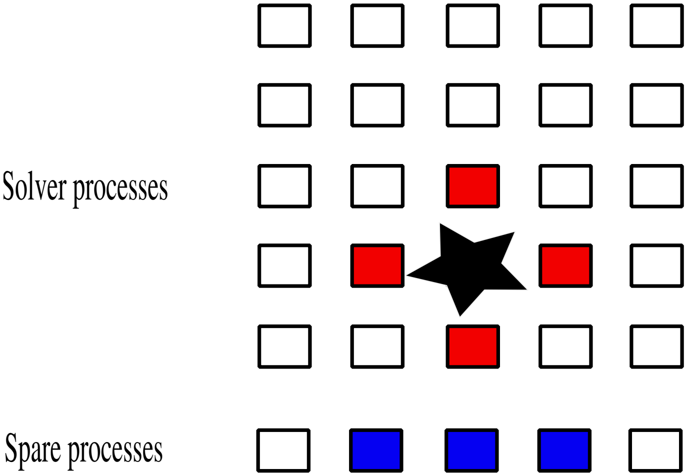
Demo XXIX

Time Step = 18 ; Checkpoint Interval = 10



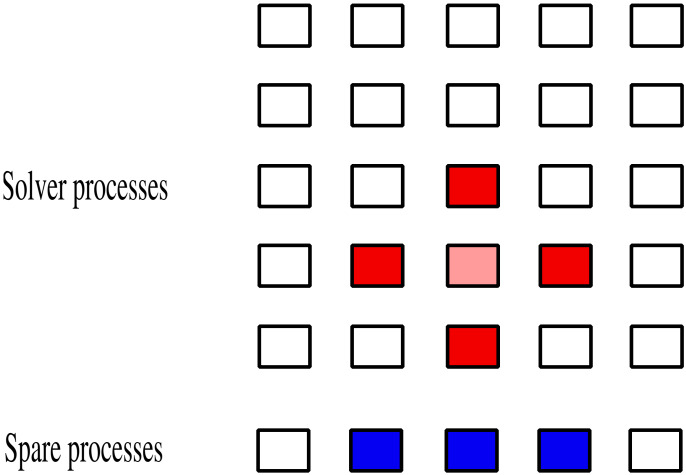
Demo XXX

Time Step = 18 ; Checkpoint Interval = 10



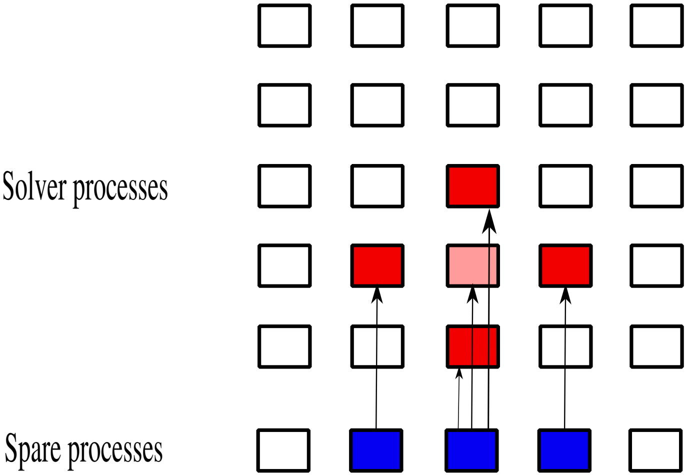
Demo XXXI

Time Step = 18 ; Checkpoint Interval = 10



Demo XXXII

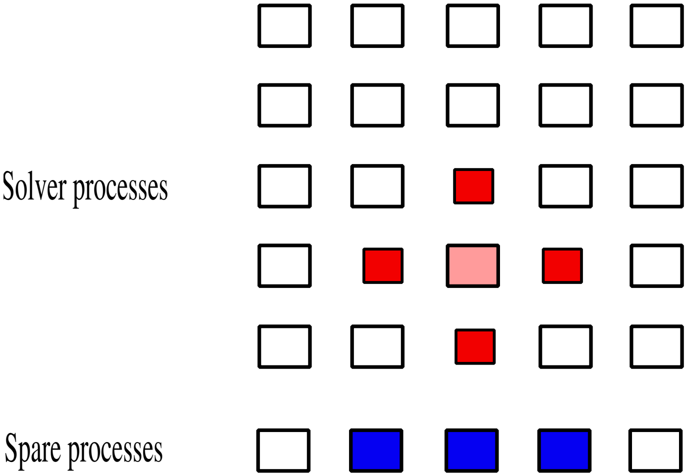
Time Step = 18 ; Checkpoint Interval = 10



Demo XXXIII

Time Step = 18 ; Checkpoint Interval = 10

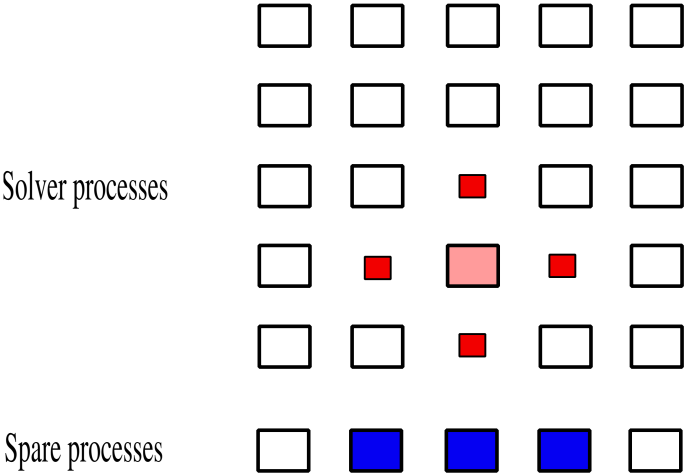
(1) Explicit Backward Integration



Demo XXXIV

Time Step = 18 ; Checkpoint Interval = 10

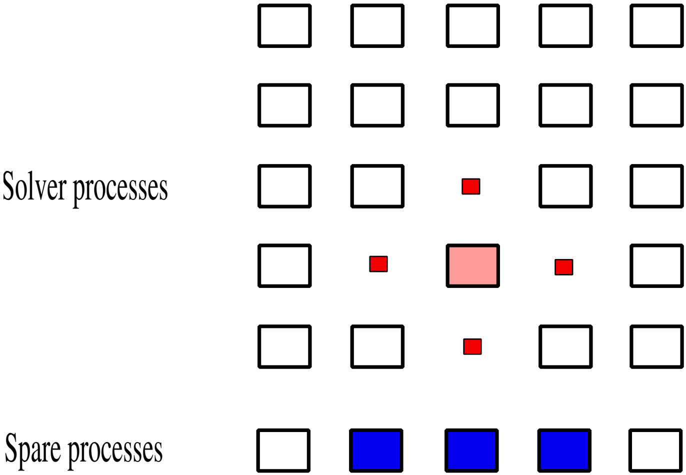
(1) Explicit Backward Integration



Demo XXXV

Time Step = 18 ; Checkpoint Interval = 10

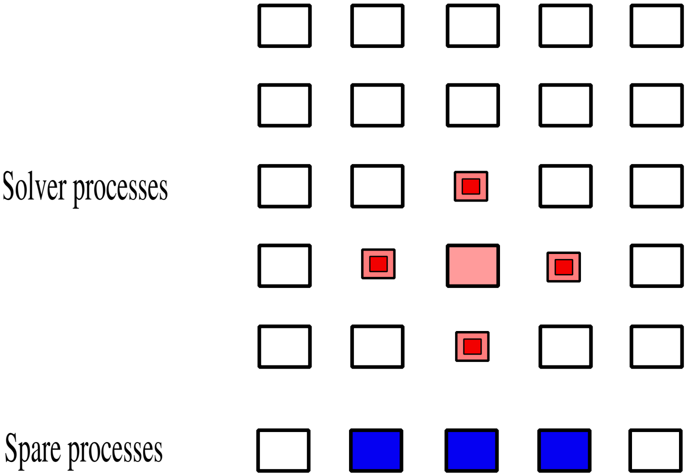
(1) Explicit Backward Integration



Demo XXXVI

Time Step = 18 ; Checkpoint Interval = 10

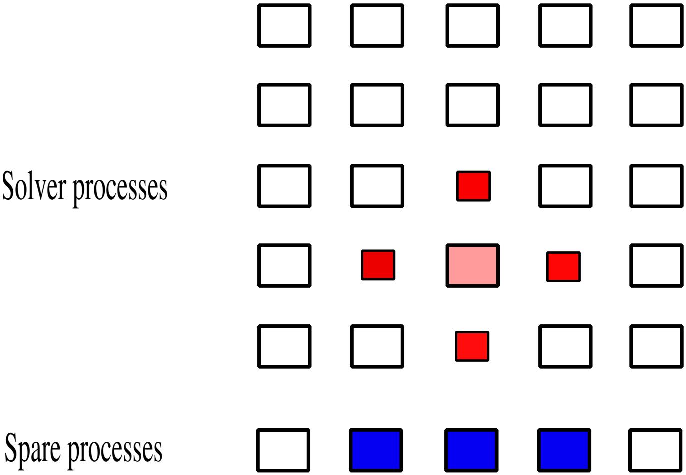
(2) Space Marching Scheme



Demo XXXVII

Time Step = 18 ; Checkpoint Interval = 10

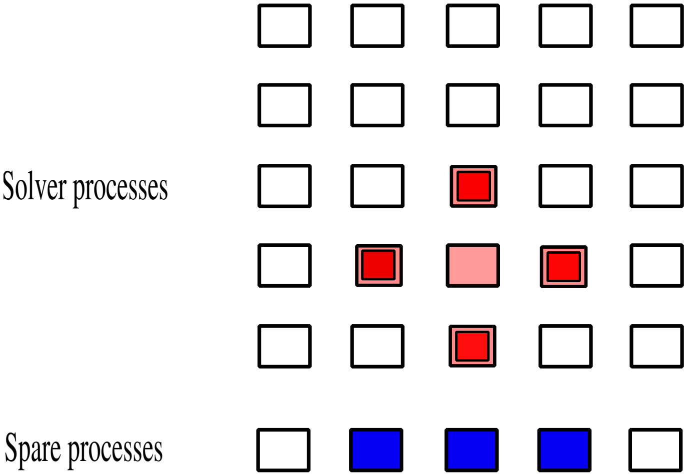
(2) Space Marching Scheme



Demo XXXVIII

Time Step = 18 ; Checkpoint Interval = 10

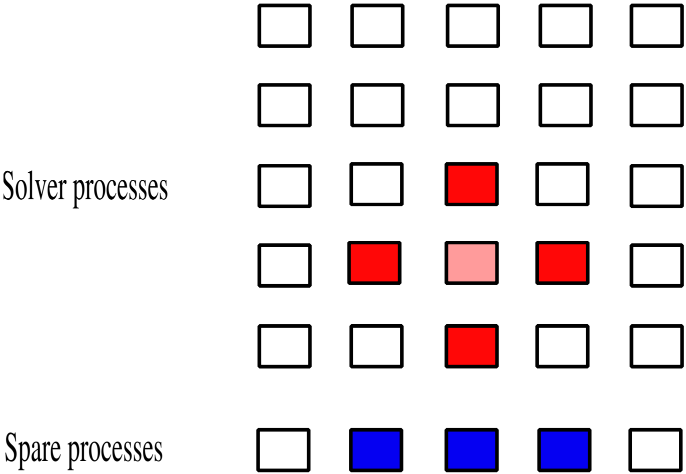
(2) Space Marching Scheme



Demo XXXIX

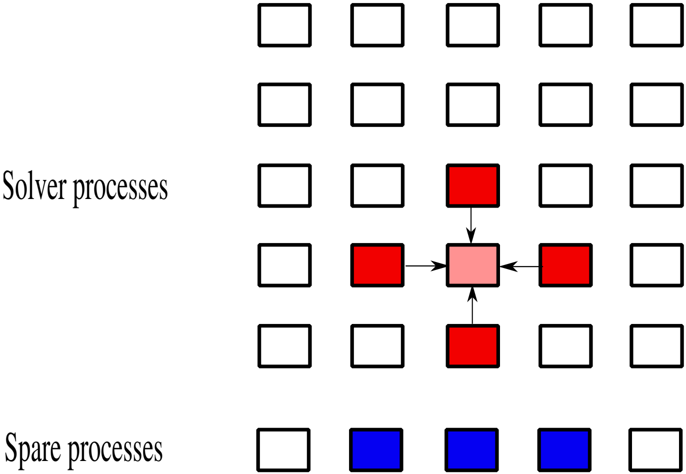
Time Step = 18 ; Checkpoint Interval = 10

(2) Space Marching Scheme

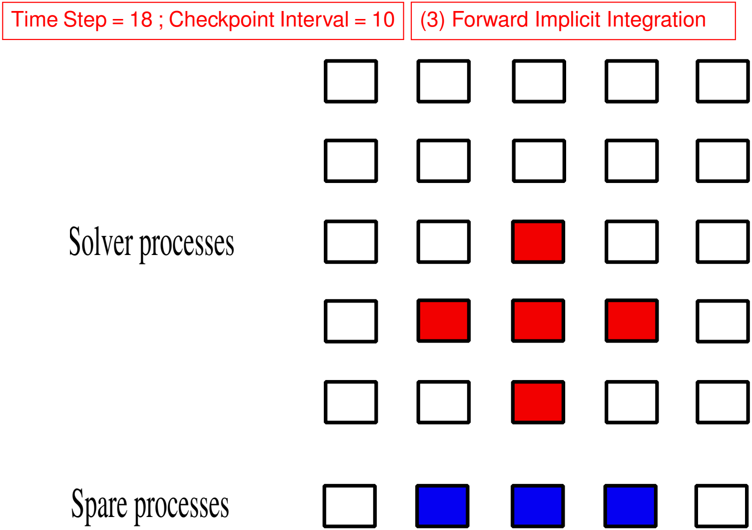


Demo XL

Time Step = 18 ; Checkpoint Interval = 10

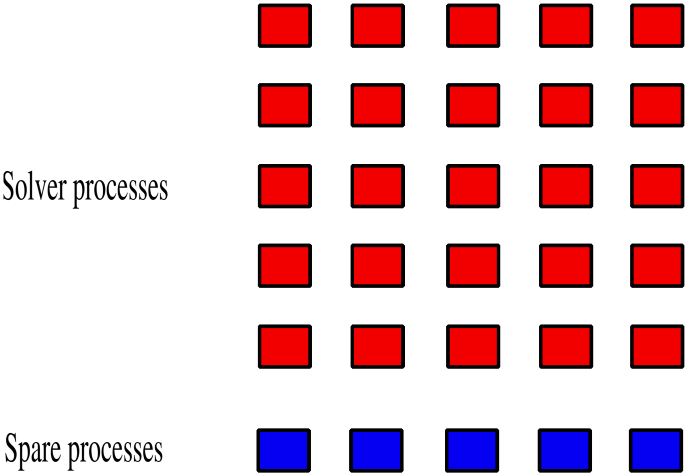


Demo XLI


Time Step = 18 ; Checkpoint Interval = 10
(3) Forward Implicit Integration
Solver processes
Spare processes

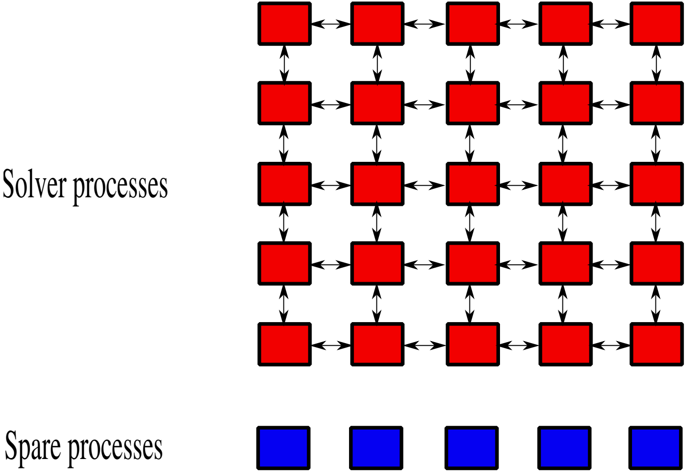
Demo XLII

Time Step = 18 ; Checkpoint Interval = 10



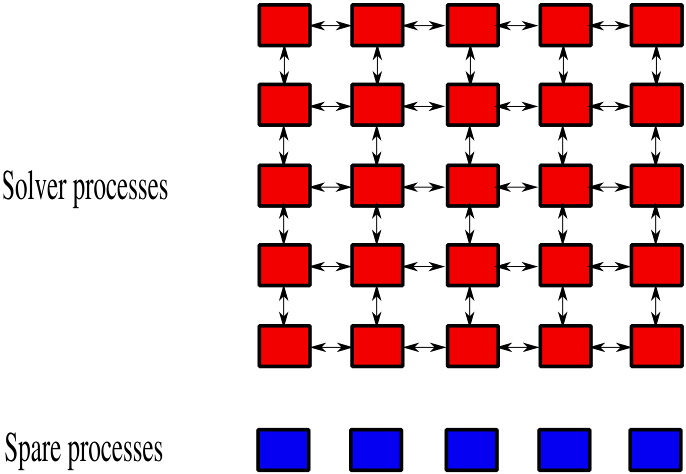
Demo XLIII

Time Step = 18 ; Checkpoint Interval = 10



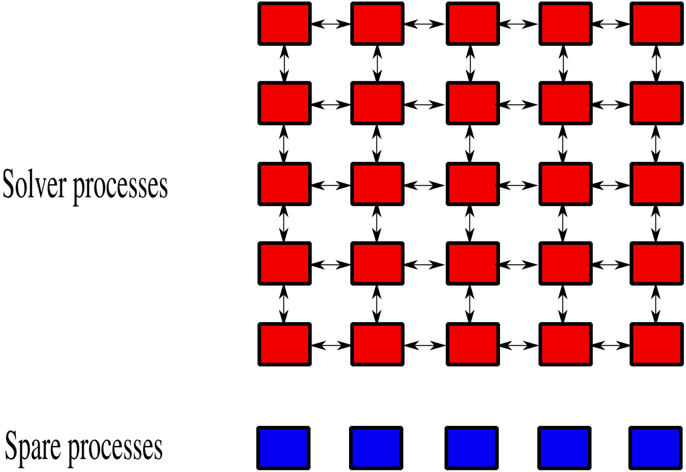
Demo XLIV

Time Step = 19 ; Checkpoint Interval = 10



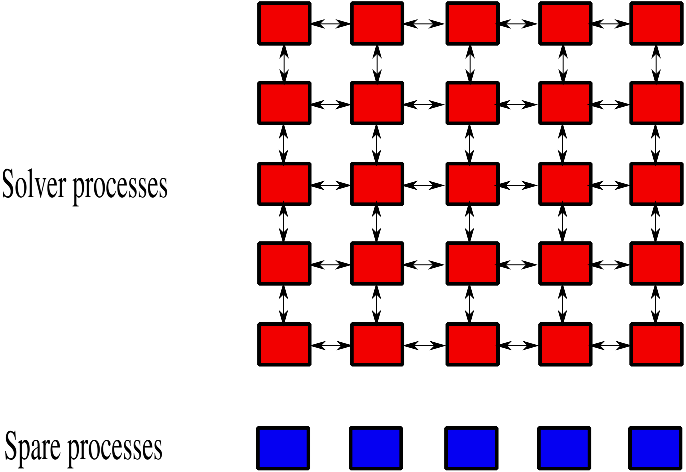
Demo XLV

Time Step = 20 ; Checkpoint Interval = 10



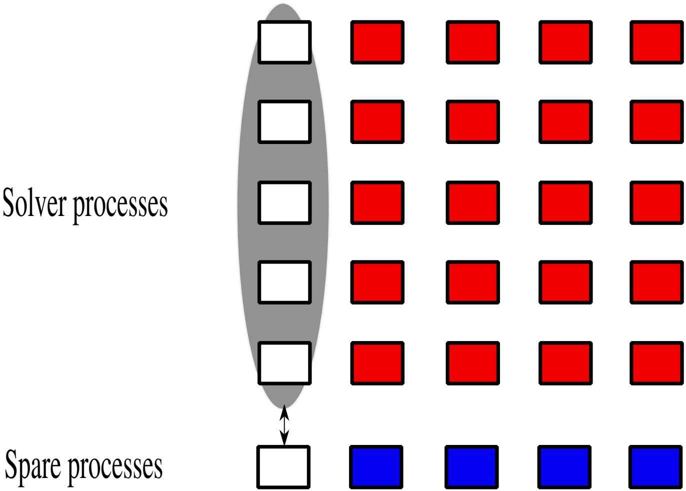
Demo XLVI

Time Step = 21 ; Checkpoint Interval = 10



Demo XLVII

Time Step = 21 ; Checkpoint Interval = 10



Results I

- Recovery Performance (A)

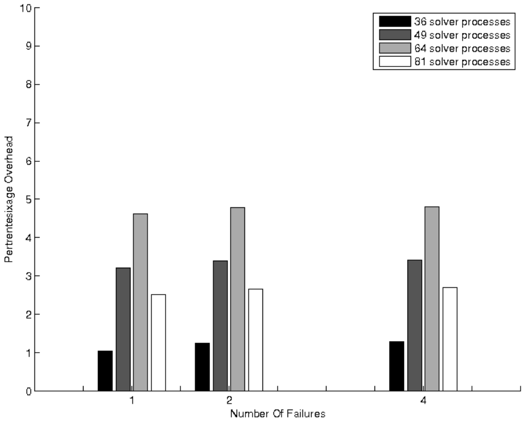


Figure: Failure Overheads in Percentage: **Small** data set.

Results II

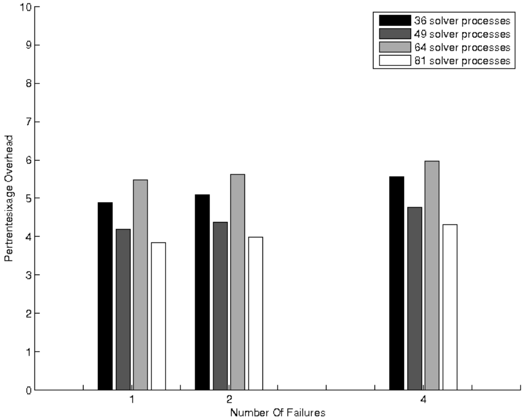


Figure: Failure Overheads in Percentage: **Medium** data set.

Results III

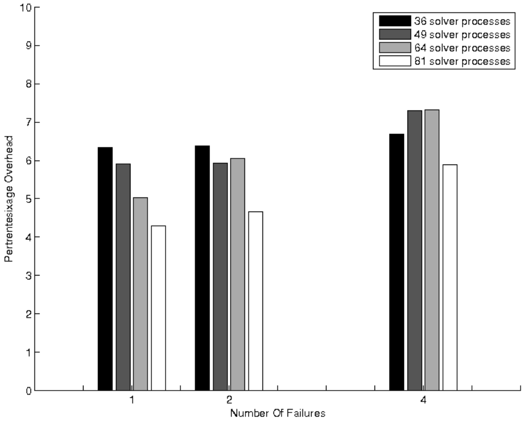


Figure: Failure Overheads in Percentage: **Large** data set.

Results IV

- Recovery Performance (B)

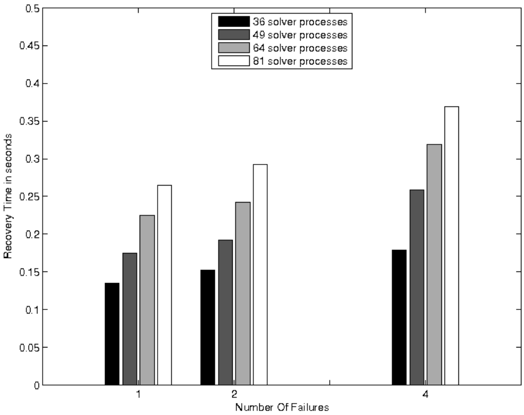


Figure: Recovery Time in Seconds: **Small** data set.

Results V

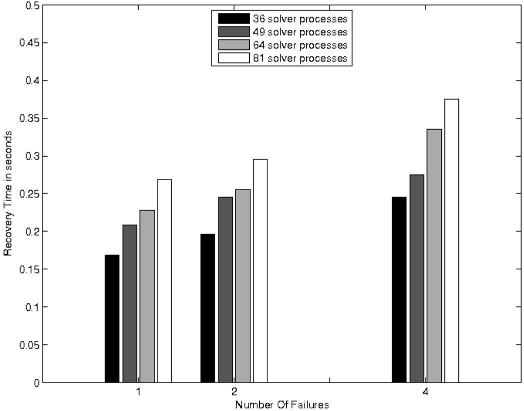


Figure: Recovery Time in Seconds: **Medium** data set.

Results VI

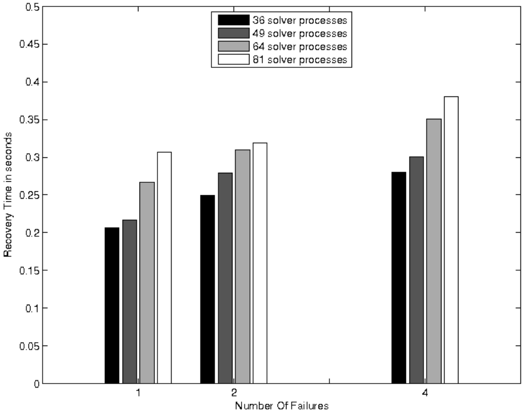


Figure: Recovery Time in Seconds: **Large** data set.

Conclusion

- Our Fault Tolerant approach is based on efficient numerical algorithms
- Solution: combine explicit reconstruction techniques that are an explicit backward integration with some stabilization terms and space marching
- Challenging problem because it is very ill posed
- One more job to do for spare processors could be solution verification *on the fly* (the least square extrapolation method)

Thank You!