On Undecidability Aspects of Resilient Computations and Implications to Exascale

Nagi Rao

(Nageswara S. V. Rao)

Oak Ridge National Laboratory

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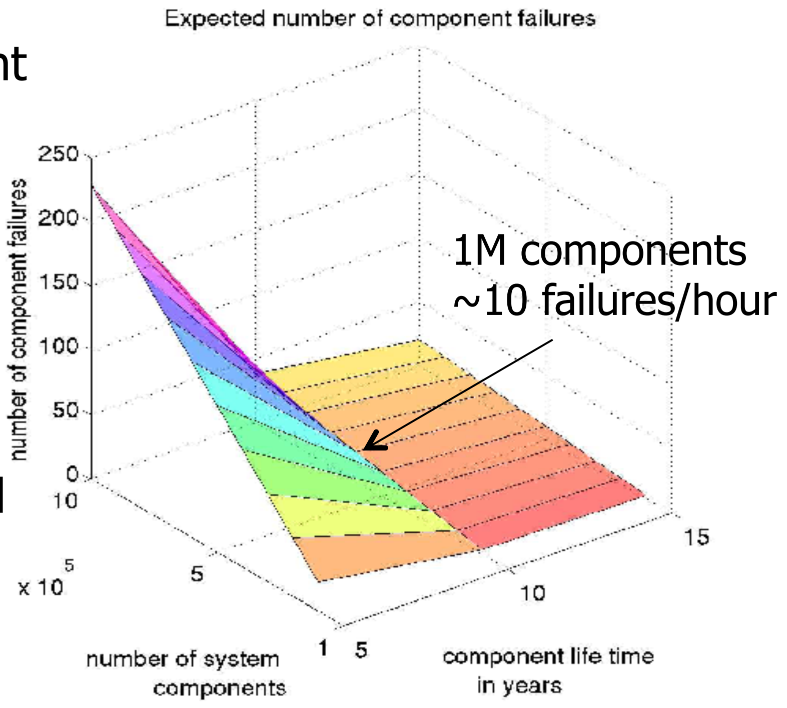
Inherent Failures in Exascale Computing Systems

- Exascale computing systems are expected to have millions of processor cores and other components.
  - components with expected life-span of ten years
    - ~100k hours/component = 10 failures/hour among 1M components
  - codes that run for a few hours likely experience failures of several components.
- Failure rates limit the effectiveness of current check-point/recovery methods:
  - Recovery times could be hours for Exascale systems
  - transient silent errors may lead to erroneous computations
- Failures will be integral part of Exascale computations – must be explicitly accounted
  - code outputs must be quantified with confidence estimates
    - specific to system failure profile
    - justifiable by measurements



Related Areas: Resilient Computations

- Foundational works:
  - von Neumann studied (in 1950s) mathematical aspects of achieving reliable computations over systems with unreliable components
  - subsequent reliability improvements in computing systems, perhaps, led to such studies not being extensively continued
- Deployed systems: computing systems in satellites
  - deployed over past decades - enhanced with Software-Implemented Hardware Fault Tolerance (SIHFT) methods to counteract errors due to radiation in space environments.

But, Exascale computations present new challenges:

- sheer size and system complexity makes dynamic profiling of the failures and robustness complicated
- computation becomes inherently probabilistic:
  - for most applications, 100% guarantee of robustness against failures in not possible
  - requires confidence measures for code outputs – running to completion is not sufficient

Faults and Computations

Fault-Free Computing System: Program P with input x is executed error free produces output $\phi_P(x) = \Phi(x, P)$

$\uparrow$ universal function or Turing Machine

P is also represented by its Godel's number $e = \# P$

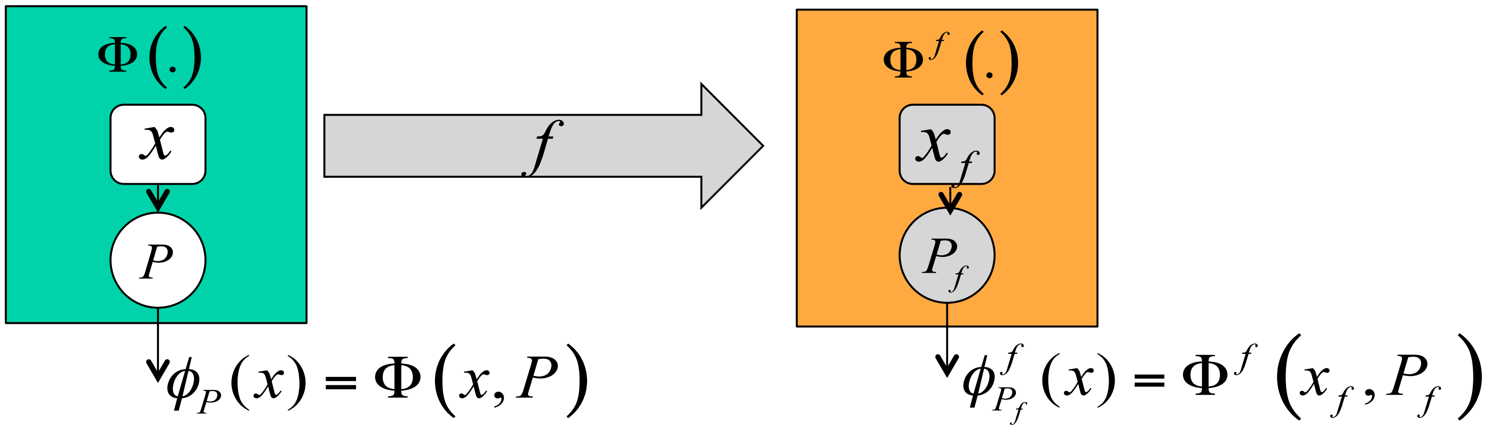
Fault: Program may crash or produce incorrect output or loop forever

Simple fault models: specified by function f

data error: x replaced by x_f

program or code error: P replaced by P_f

execution error: $\Phi(.)$ replaced by $\Phi^f(.)$



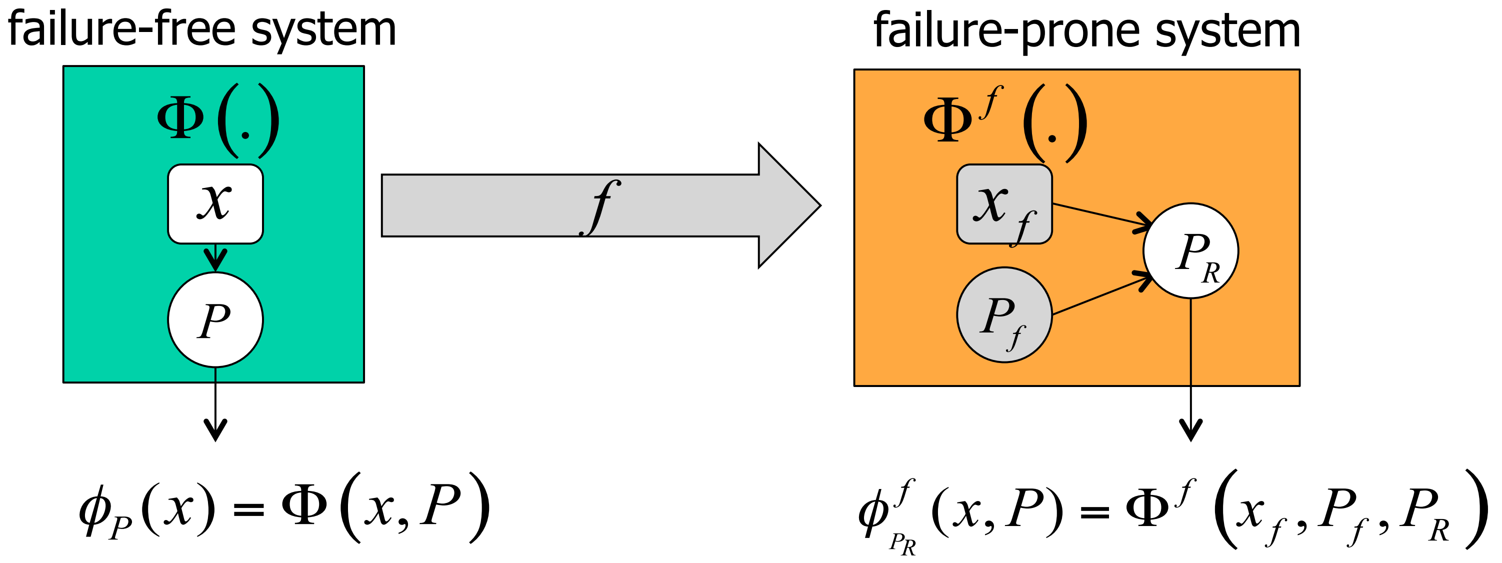
Simplified models: specified by separate functions $f = (f_D, f_P, f_E)$

$f_D : x \rightarrow x_f \qquad f_P : P \rightarrow P_f \qquad f_E : \Phi \rightarrow \Phi_f$

Faults may be more complicated: stochastic, correlated, multi-valued

Faults and Resilient Computations

Resilient Version of Program P: Another program P_R on fault-prone system with same output and halting properties as P on failure-free system



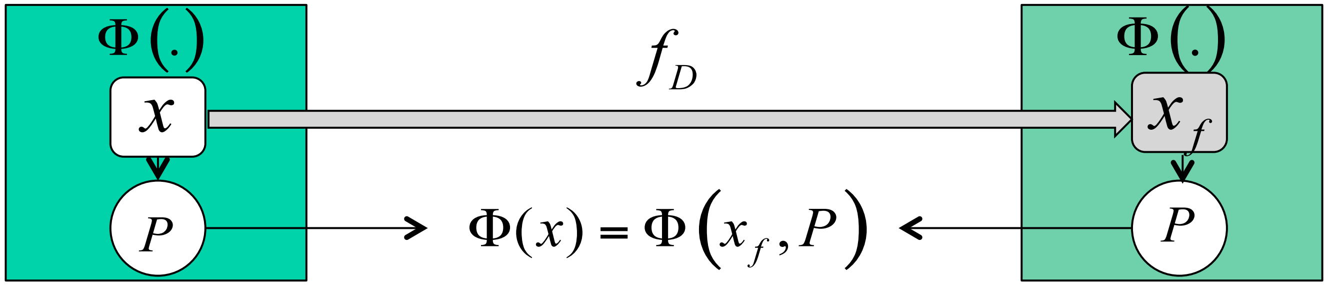
Requirement: Resilient version P_R of the program P is required to produce on the failure-prone system $\Phi^f(.)$ same output as original program on the failure-free systems: that is,

$$\phi_P(x) = \phi_{P_R}^f(x, P)$$

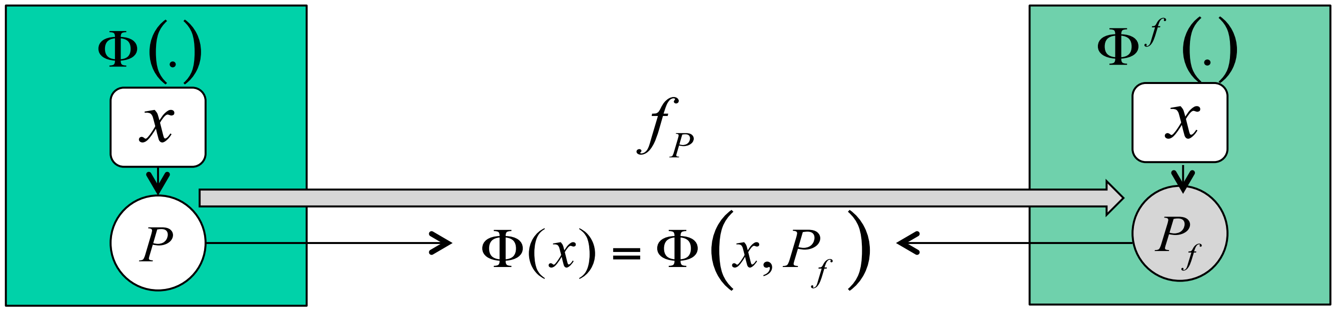
$$\Phi(x, P) = \Phi^f(x_f, P_f, P_R)$$

Individual Data, Program and Executional Faults

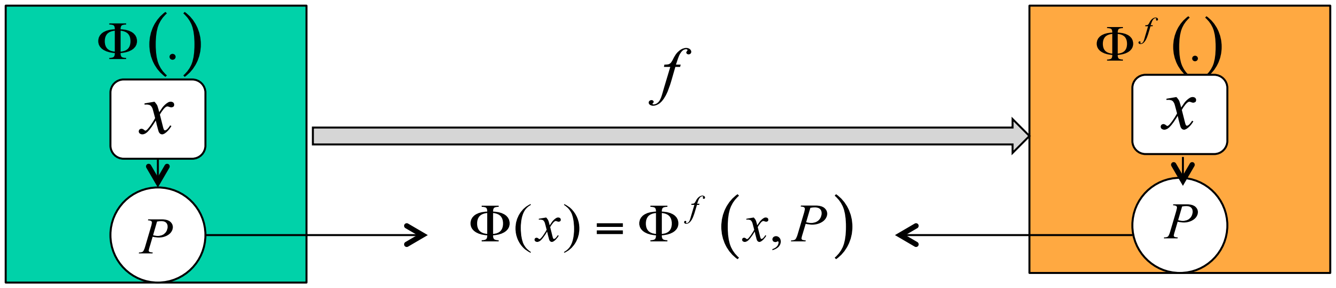
(a) Data faults: $f_D : x \to x_f$



(b) Program faults: $f_P : P \to P_f$



(c) Executional Faults: $f_E : \Phi \to \Phi_f$



Context of Undecidability

Turing's Undecidability or Uncomputability: Non-existence of algorithms for certain class of computing problems

- These are "harder" than NP-hard problems
- Examples: halting problem, loop detection problem, equivalence of context-free grammar, virus detection problems
- **Resilient Computations**: Certain resilient computational problems turn out to be undecidable

Godel's Undecidability: Non-existence of mathematical proofs for true statements within an axiomatic system such as arithmetic system with multiplication

- Arithmetic systems with addition and multiplication operations have statements that are true but cannot be proved as theorems
- **Resilient Computations**: Certain resilience properties of computer codes cannot be proved

These two results are closely related: algorithmic information theory of Gregory Chaitin

Resilient computations: certain faults may embody information beyond the capacity of axiomatic system or finite computations

- not be compensated purely by mathematical or computing means
- but, may be handled by other means, such as fault monitors

Complex Faults in Exascale Systems

1. **Code Corruption**: Executables may be corrupted and lead to infinite loops
   - condition $i < N$ may be changed to $i > 0$
   - go to statements changed to be self-referential
2. **Errors in Parameters and Variables**: infinite loops can be created by corrupted variable values
   - changing boundary values of loop variables
3. **ALU Circuit Errors**: arithmetic and logic execution errors can create infinite loops
4. **Program Counter Errors**: loading errors in program counters will lead computations directed to unexpected locations

Composite Errors: Hot-spots in server cabinets lead to overheating – that effects multiple parts of the system

Language L and Turing machines: codes and data both appear as strings

memory errors effect them both

Resilient Computations

Under Programming Language L

Program P is consists of instructions

Converted to numerical code $y = \#(P)$ using Godel numbering

$\phi_y(x)$ is output of program

Universality Theorem: there exists a universal function $\Phi(.)$ such that

$$\Phi(x, y) = \phi_y(x)$$

Resilient version: y_R is resilient version of y

Functions computed on fault-prone machine: with input x

original non-resilient program: $g(x)$

resilient-version: $g_R(x)$

Set of original programs: A

Examples: non-linear solvers, climate codes, …

All their resilient versions: $R = \left\{ y_R \mid y \in A \right\}$

index set:

$$R_R = \left\{ t \in N \mid \phi_t \in R \right\}$$

Undecidability of Verification of Resilient Computations

Resilience Property is not verifiable computationally:

index-set $R_R = \{t \in N \mid \phi_t \in R\}$ is not computable – **Theorem 1**

Proof outline: By contradiction – assume it is computable

$$h(t,x) = \begin{cases} g(x) & if \quad t \in R_R \\ g_R(x) & if \quad t \notin R_R \end{cases}$$

If R_R is computable, then the following function is partially computable

$$h(t,x) = 1_{R_R}(t) g(x) + \left[1 - 1_{R_R}(t)\right] g_R(x)$$

where $1_S(.)$ is the indicator function of set S such that

$$1_S(x) = \begin{cases} 1 & if \quad x \in S \\ 0 & if \quad x \notin S \end{cases}$$

By recursion theorem, there exists a program *e* such that

$$\phi_e(x) = h(e,x) = \begin{cases} g(x) & if \quad \phi_e \in R \\ g_R(x) & if \quad \phi_e \notin R \end{cases}$$

Undecidability of Verification of Resilient Computations

Proof outline continued: by contradiction

By universality theorem, there exists a function $\Phi(.,.)$ such that

$$\Phi(x, e) = \phi_e(x)$$

Question: which set does *e* that computes $h(e, x)$ belongs to?

Answer is neither as shown below

Case A: it is resilient code $e \in R_R$

implies on error-prone system: by definition of *h(.)*

$$\phi_e(x) = h(e, x) = g(x)$$

but its function is same as original non-resilient $g(x)$

Case B: it is non-resilient code $e \notin R_R$

implies on error-prone system: by definition of *h(.)*

$$\phi_e(x) = h(e, x) = g_R(x)$$

but its function is same as resilient version

Resilient Computations and Halting Problem

Halting Problem: HALT(x, y) is true if and only if y halts on input x

- it is well-known to be undecidable

Loop Detection Problem: NO-LOOP(x, y) is true if and only if y does not loop indefinitely on input x

Fault Function: $f(x, y) = (x_f, y_f)$

replace both data and code by faulty versions

- it is only a special abstracted case of faults

Resilience Versions: RESILIFY(x, y, f) is true if and only if there exists a program P_f for failure-prone system that:

takes input x_f and produces the same output as y with input x on failure-free system

Uncomputability of RESILIFY(x, y, f)

Loop Creating Faults:

RESILIFY(x, y, f) is not a computable predicate – Theorem 2
Proof outline: Equivalent to RESILIFY(x_f, y_f, I) where I is the Identity function that creates no faults

input and code are changed to x_f and y_f instantaneously

It is equivalent to NO-LOOP(x_f, y_f), which is non-computable:

Consider the program:

```
[P]: if  NO-LOOP( $x, x$ ) go to  P
         else return NO
```

index of the code $y_0 = \#P$

NO-LOOP(x, y_0) is true if and only if NO-LOOP(x, x) is false

Using $x = y_0$ we have contradiction

NO-LOOP(y_0, y_0) is true if and only if NO-LOOP(y_0, y_0) is false

Resilient Computations on Turing Machines

A Turing Machine consists of:

- infinite tape of cells – read/write head that process them one at a time, and moves left or right or no motion
- Finite set of states that control the computation

TM *M* starts with input *w* on the tape, and goes through state transitions and writes output on to the tape

Same computation is done by Universal TM (UTM) by specifying both *M* and *w* as input on tape

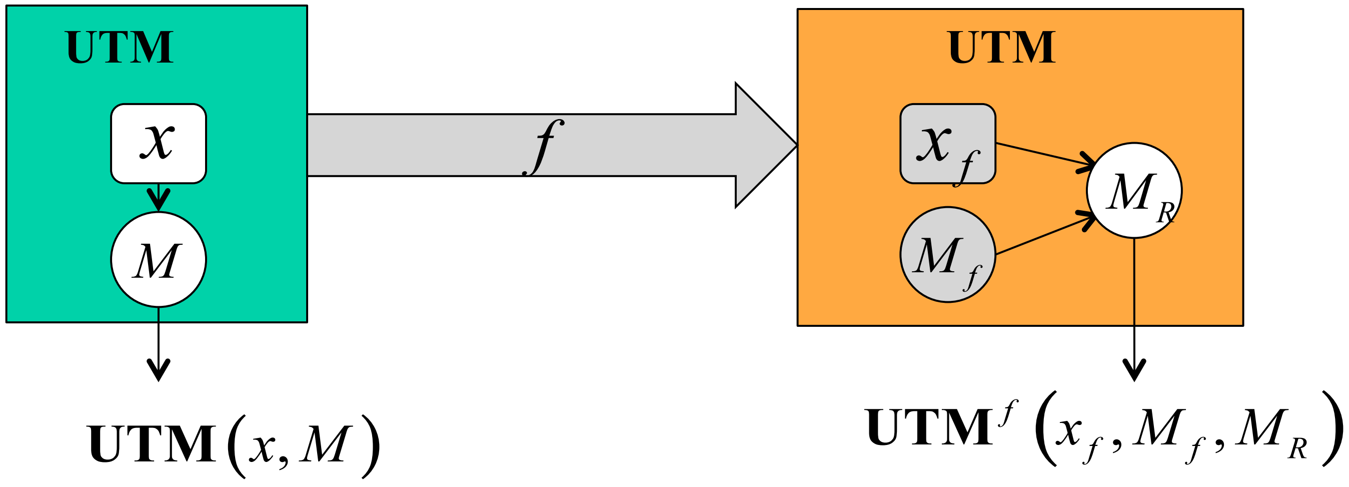
Faults are more "concretely" visualized electro-mechanically on TM:

- **memory faults**: contents of cell may be corrupted
- **code errors**: states may be corrupted
  - more easily see if code is written on tape and executed by UTM
- **execution errors**:
  - state transition faults – analogous to program counter errors
  - read/write head errors – analogous to memory to register transfer errors

Failure-prone version of M is denoted by M_f under fault function f

Faults and Resilient Computations on TM

Resilient Version of TM M: Another TM M_R on fault-prone system with same output and halting properties as M on failure-free system



Requirement: Resilient version M_R of TM M is required to produce under the failure conditions the same output as the original failure-free TM:

$$\mathbf{UTM}(x, M) = \mathbf{UTM}^f(x_f, M_f, M_R)$$

Resilient Computations on Turing Machines

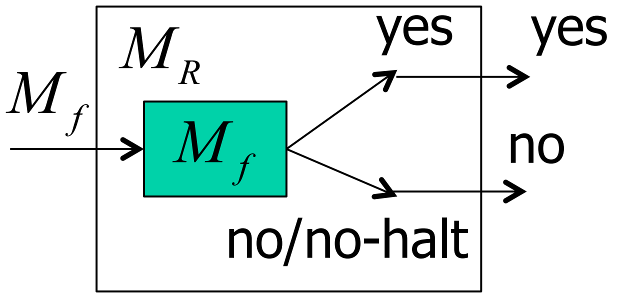
Resilient Turing Machine: M_R operates under fault function f

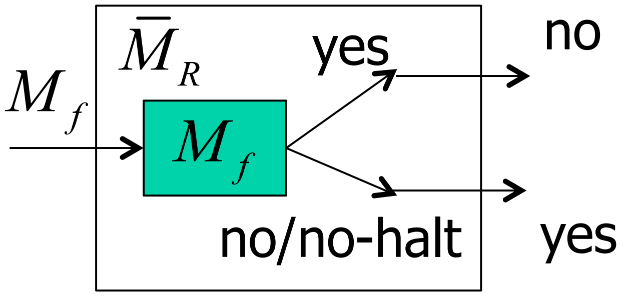
- RTM takes as input TM M and its input w
- halts and produces same output as M when executed by failure-free UTM with input w

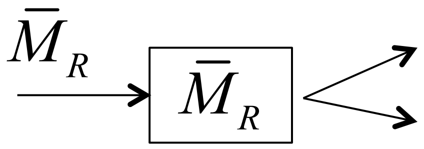
Resilient Turing Machines do not exist under executional and data failures – Theorem 3

Proof Outline: If M_R exists: it detects when M_f does not halt, and interject and produces failure-free output of M

Halting problem:







Yes => $\bar{M}_R$ outputs no or does not halt

No => $\bar{M}_R$ outputs yes

Difficulties of Mathematical of Resilience Proofs

Simple Computing Example: Compute $G(x,a) = x^a$

Computations are limited to multiplications, additions and these functions – all are integer computations

Computing Errors: limit magnitude of operand to $x_f < x$

– zeroing of higher-order bits in registers: it computes $G(x_f, a)$

Potential Solution: program that makes up the difference by computing and adding the "rest" of the value such that:

$$G(x,a) = G(x_f, a) + G(y,a)$$

Difficulty: Proof of this resilience property requires us to contradict Fermat's last theorem that claims no solution exists for $a > 2$:

$$x^a = x_f^{\ a} + y^a$$

Proof has remained open for 300 years until 1995 by Andrew Wilks

Illustration of difficulties in proving a rather "simple" resilience properties

Beyond Halting Problem: Relativized Computations

Foundational Question: Does solving the Halting problem give us complete resilience?

Short answer: most likely NO – certain complex faults may remain that still lead to undecidable problems

Relativized Complexity Classes: Model Halting Problem solution as an oracle to abstract away the complexity:

> Relativized Rice Theorem establishes that assessing general resilience properties of computations remains undecidable under the Halting Problem Oracle.

Resilient Computations:

- Halting problems address classes of problems related to creation of infinite loops
- Ensuring that outputs of resilient versions are identical to output of original codes poses additional challenges somewhat analogous to checking equality of context-free grammars:
  - yet another class of undecidable problems

Conclusions

Addressed computational aspects of resilient computations under broad class of faults

Resilient computations present significant computational challenges:

(a) asserting resiliency of computations is non-computable

(b) mathematical proofs of resilience of algorithms are undecidable

These problems are not solvable in general form by computations and mathematical proofs alone: but,

- resilient computations can be designed for specific classes
- additional fault detection methods could make some problems computable

In general, these results motivate: deeper investigations of fault classes and resilient computations customized for them with complementary information

Future Work:

- These results are only a very first step
- Deeper study of fault classes in needed
  - impact on computations and resilience mechanisms
- Study of hierarchy of undecidable problems and corresponding faults (vice versa):
  - is there a taxonomy of faults that reflects this hierarchy
    - seems reasonable to expect: some fault are harder to protect against

Thank you