Multilevel Monte Carlo Method with Application to Uncertainty Quantification in Oil Reservoir Simulation

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Problem Statement

Uncertainty quantification in reservoir simulation

- A 3-D reservoir model with size 60’x20’x85.
- The permeability field is strongly heterogeneous.
- Quantification of the influence of the permeability uncertainty on oil and gas production is very computationally expensive.
- The rational management of oil and gas reservoir requires an understanding of its response to existing and planned schemes of exploitation and operation.
- Such understanding requires analyzing and quantifying the influence of the subsurface uncertainty on predictions of oil and gas production.
- In uncertainty quantification, the commonly used stochastic techniques like moment equation methods, generalized polynomial chaos expansions and stochastic collocation methods are not suitable for the strongly heterogeneous reservoir problem.
- The dimension independent Monte Carlo simulation becomes the choice.

Monte Carlo simulation

- Based on measurements of permeability $k$, random realizations of $\log k$ are generated.
- For each realization of $\log k$ field, simulate the reservoir model to get simulations of oil production $Q$.
- Summarize simulated results of $Q$ statistically to quantify the influence of $\log k$ uncertainty on $Q$.

Computational cost

- The standard MC simulation is very computationally expensive because
  - a large number of model executions is required to achieve convergence;
  - each model execution is time costly simulated on a fine spatial grid to ensure accuracy.
- A computationally efficient method is desired to quantify the uncertainty of the strongly heterogeneous reservoir problem.

Standard Monte Carlo Method

We are interested in estimating the expectation $E(Q)$ of predicted oil production $Q$ by simulating $Q$ at the numerical model with size $M$.

The standard MC estimator for $E(Q_d)$ is

$$E(Q_d) = \hat{Q}_d^{MC} = \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} Q_d^{(i)}$$

The mean square error (MSE) of the estimator is

$$e(\hat{Q}_d^{MC})^2 = E\left(\left(\hat{Q}_d^{MC} - E(Q)\right)^2\right) = N_{MC}^{-1} \sum_{i=1}^{N_{MC}} (Q_d^{(i)} - Q_d)^2$$

- First term is variance of the MC estimator, which is small as $V(Q_d)$ is small and decays inversely with the number of samples $N_{MC}$.
- Second term is square of the error in expectation between $Q_d$ and $Q$, which can be reduced by using a fine numerical grid.

Multilevel Monte Carlo Method

The idea of MLMC can be formulated as:

$$E(Q_M) = E(Q_M) + \sum_{l=0}^{L} E(Q_{M_l}) = \sum_{l=0}^{L} E(Y_l)$$

where $l=0, 1, \ldots, L$ is levels, and model grid size $M_{l<}M_{l<}M_l=M$.

$$E(Y_l) = \hat{Y}_l = \frac{1}{N_l} \sum_{i=1}^{N_l} (Q^{(i)}_l - Q^{(i)}_{l-1})$$

The MLMC estimator for $E(Q_d)$ is

$$E(Q_M) = \hat{Q}_M^{MLC} = \sum_{l=0}^{L} \hat{Y}_l$$

The MSE of the MLMC estimator is

$$e(\hat{Q}_M^{MLC})^2 = E\left(\left(\hat{Q}_M^{MLC} - E(Q)\right)^2\right) = L \sum_{l=0}^{L} N_l^{-1}V(Y_l) + (E(Q_M) - E(Q))^2$$

- First term reflects the advantage of MLMC estimator achieving the same accuracy as standard MC but with less computational cost.

Application to a Reservoir Model

Model description

- (a) True permeability field $\log(k)$;
- (b) 36 sample data conditioned to generate $(\log(k))$ realizations;
- (c) One realization of $\log(k)$ field.

We are interested in estimating oil production $E[Q]$ at location $P$.

Implementation of MLMC

- Six levels used in MLMC

Variance decays with levels

- $h_0$ is cell length of the coarsest level; $\lambda$ is correlation length.
- The coarsest level is determined as $h_0 < \lambda$.
- Variance decreasing with levels suggests fewer samples are needed on computationally costly higher levels.

Computational efficiency of MLMC

- To achieve the same RMSE, MLMC needs significantly less computational time.
- For the same computational time, MLMC can achieve higher accuracy with smaller RMSE.

Conclusions

- MLMC method can evaluate uncertainty in strongly heterogeneous reservoir problems efficiently and effectively.
- MLMC method is model independent and flexible to be used together with any MC estimators.