Modeling the Fracture of Ice Sheets on Parallel Computers

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Motivation: Importance of Ice Fracture

Objective: Employ parallel computers to study the fracture of land ice to better understand how it affects global climate change. In particular the collapse mechanism of ice shelves, the calving of large icebergs and the role of fracture in the delivery of water to the bed of ice sheets.

Example 1 global warming leading to collapse of ice shelves in Antarctica

Larsen B diminishing shelf 1998-2002

Wilkins ice shelf Recent 2008 collapse
Motivation: Importance of Ice Fracture

Example 2: ice calving from an ice shelf

Fracture and disintegration to smaller Icebergs

The proposed research will be used to validate theories, for example:
Alley et. al., Science [2008]: a simple law for ice shelf calving.
Motivation: Importance of Ice Fracture

Example 3 role of fracture in delivery of water to the bed of ice sheets

_Disappearance of huge Supraglacial lakes in minutes_

**Explanation:** lakes build up pressure that cracks the ice sheets, creating conduits that penetrate the ice and eventually drains to the bottom (base) of ice sheet. Sometimes referred to *Ice Sheet’s Plumbing System*

**Main Consequence:** the water driven fracture flows directly to the base of the ice sheet, raising the surface of the ice sheet (orders of meters) and lubricating its base. The net effect is that the ice sheet flow is accelerated due to this lubrication.
Proposed research
(strong ties with DOE programs: TOPS, CSCAPES)

Exploration: examine/learn and evaluate existing ice sheet models as base code development platform and seek new partners in the ice and climate communities

Problem definition: define geometry (from terrain data), boundary condition and loads (self weight) and generate a mesh

Modeling: continuum damage mechanics (crack initiation and propagation), elasticity and extended finite element (XFEM) for crack modeling

Solution: developing specialized highly parallel multigrid solvers for XFEM

Verification & Validation of the code with available experimental data
Modeling Step

Damage Mechanics (crack initiation)

\[ \sigma_{ij}^{\text{eff}} = (1 - D)\sigma_{ij} = (1 - D)C_{ijkl}\epsilon_{kl} \]

\[ 0 < D < 1 \]

**Lemaitre Damage Law:**

\[ D(\varepsilon) = 1 - (1 - A)\varepsilon_{D0}\varepsilon^{-1} - Ae^{-B(\varepsilon - \varepsilon_{D0})} \]

Generalized Thermo-Elasticity (ice modeling)

\[ \nabla_s^T \sigma + b = 0 \quad \text{in} \quad \Omega \]
\[ \sigma = D\varepsilon \quad \text{in} \quad \Omega \]
\[ u = \bar{u} \quad \text{on} \quad \Gamma_u \]
\[ \tau \cdot n = \bar{t} \quad \text{on} \quad \Gamma_t \]

Extended Finite Elements (crack modeling)

\[ u^h(x) = \sum_{I=1}^{n} N_I(x)u_I + \sum_{I=1}^{n_J} N_I(x)H(x)a_I + \sum_{I=1}^{n_T} \left[ N_I(x) \sum_{j=1}^{4} F_j(x)b_{jI} \right] \]
Briefly: How XFEM works

The displacements field in XFEM is written as

\[ u^h(x) = \sum_{l=1}^{n} N_l(x)u_l + \sum_{l=1}^{n} N_l(x)H(x)u_l + \sum_{l=1}^{nT} \left[ N_l(x) \sum_{j=1}^{4} F_j(x)b_{jI} \right] \]

Near-tip asymptotic enrichment functions

\[ \{F_i(r, \theta)\} = \left\{ \sqrt{r} \sin \left( \frac{\theta}{2} \right), \sqrt{r} \cos \left( \frac{\theta}{2} \right), \sqrt{r} \sin \left( \frac{\theta}{2} \right) \sin(\theta), \sqrt{r} \cos \left( \frac{\theta}{2} \right) \sin(\theta) \right\} \]

Heaviside jump-enrichment

\[ H(X) = \begin{cases} 1 & \text{above } \Gamma^+ \\ 0 & \text{below } \Gamma^- \end{cases} \]

Fracture in 3D

Levelset Method

Belytschko et. al., IJNME [1999]
Modeling Fracture with XFEM

Modeling holes and cracks

- FEM requires special meshes to model flaws
- XFEM requires special enrichments but regular meshes to model flaws

H. Waisman, E. Chatzi and A. Smyth, accepted, IJNME [2009]
Crack Propagation

Solution Step

Advantages of XFEM
provides modeling flexibility and can be used to
1. Predict fracture and collapse of ice sheets
2. Predict ice calving
3. Explain accelerated ice sheet flow

Drawback of XFEM
adds degrees of freedom and accurate modeling will quickly result in

Billion of Unknowns

Need Efficient Parallel Solver

Strong ties with SciDAC TOPS program: Toward Optimal Petascale Simulations

D. Keyes et. al. Implicit solvers for large-scale non-linear problems. JCP [2006]
Observation from classical iterative methods:
Multigrid Principles

- **smoothing**
- **R** restriction operator
- **P** prolongation operator
- Coarsest scale: direct solve

Repeat until convergence

V-cycle

**Research**

Multigrid for XFEM is not trivial and traditional methods may not converge since cracks are embedded in the matrix (special formulation is needed).

We will employ the *ML* package of the *Trilinos* project (*TOPS*)

M. Heroux et. Al., An overview of the Trilinos project, ACM Transactions on Mathematical Software 2005
Acceleration techniques for MG-XFEM

filtering cycle error:

\[ e^{i+1} = (S^u T^i_{MG} S^v) F_{GGB} (S^u T^i_{MG} S^v) e^i \]

H. Waisman, J. Fish, R. Tuminaro and J. Shadid, IJNME [2004]
Parallel Computing and Load Balancing

We will employ *DOE ASCR leadership class* computers

- Thousands of cores

Data (work) will be assigned to processors such that

- The work is evenly balanced among processors
- Communication is minimized

Use the *Zoltan* toolkit (by CSCAPES: Institute for Combinatorial Scientific Computing and Petascale Simulations)

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Load Balancing Challenges

- **Enrichment functions in XFEM**
  - Elements are not all equal
  - Use *weights* to represent work at each node or element

- **Dynamic problem**
  - New cracks nucleate at different locations
  - Cracks may propagate, branch, coalesce and have complex behavior

**Need special dynamic load balancing algorithms throughout the simulation**

*Research*

Strong ties with **SciDAC** (CSCAPES: Institute for Combinatorial Scientific Computing and Petascale Simulations)
Summary

• This research will result in a computational tool on parallel supercomputers that will increase our understanding of the role of fracture in the mechanics of polar ice sheets

• It is critical that we interact and learn from the ice modeling community so that we understand how a fracture modeling capability would be of most use

• A broader potential impact is the study of fracture and microcracking in many other engineering applications