Functions

Exercise 8.1 (Properties of functions) We may categorise the nine elements of $\{0, 1\} \nrightarrow \{0, 1\}$ according to whether they are injective, surjective, both, or neither. State which of these nine functions are:

(a) total

(b) neither injective nor surjective

(c) injective but not surjective

(d) surjective but not injective

(e) bijective

□

Exercise 8.2 (Overriding and restriction) If f and g are elements of the set of partial functions $X \nrightarrow Y$ and A is a subset of X, show that

$$A \lhd (f \oplus g) = (A \lhd f) \oplus (A \lhd g)$$

□

Exercise 8.3 (Injectivity, image, and intersection) If f is an element of injective functions $X \rightarrowtail\!\!\!\!\rightarrow Y$, and A_1 and A_2 are both subsets of X, show that

$$f(\!| A_1 \cap A_2 |\!) = f(\!| A_1 |\!) \cap f(\!| A_2 |\!)$$

□

Exercise 8.4 (Approximation) If i is an integer, and A is a set of integers, then $\mathit{closest}\ i\ A$ is an integer in A such that the absolute value $|\ \mathit{closest}\ i\ A - i\ |$ is as small as possible. Use an axiomatic definition to introduce *closest* as a curried function from integers, to sets of integers, to integers. □