TWO-PHASE FLOW WITH RANDOM PERMEABILITY BASED ON THE TENTH SPE COMPARATIVE MODEL

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We consider a two-dimensional two-phase immiscible flow in a heterogeneous porous subsurface region. For a system of two immiscible fluid phases that each consists of a single component, i.e., oil and water, we write one conservation equation for each phase,

$$\frac{\partial}{\partial t}(\phi \rho_\alpha S_\alpha) + \nabla \cdot (\rho_\alpha \mathbf{v}_\alpha) = 0, \quad \alpha \in \{w, o\}, \tag{0.1}$$

where the subscripts w and o denote the water and oil, respectively, ρ_α, $\mathbf{v}_\alpha$ and S_α denote the density, velocity and saturation of the phase α, and ϕ is the porosity of the subsurface media. To form a closed model, the Darcy's law can be extended to the two-phase flow case by using the concept of relative permeabilities. As such, the velocities $\mathbf{v}_\alpha$ for $\alpha \in \{o, w\}$ can be represented by

$$\mathbf{v}_\alpha = -\frac{K(\omega) k_{r\alpha}(s_\alpha)}{\mu_\alpha(p)} \nabla p, \tag{0.2}$$

where p is the pressure, the relative permeabilities $k_{r\alpha}$ may depend on the saturations, and the fluid viscosities $\mu_\alpha(p)$ may depend on the pressure. The randomness is in the absolute permeability $K(\omega)$, where we assume the porous media is isotropic.

Now we describe the definition of the random permeability field $K(\omega)$. The random field is constructed based on the assumption that we only have a few measurements of the true deterministic permeability field. In this problem, we use one layer of the permeability data of the model 2 in The 10th SPE Comparative Solution Project http://www.spe.org/web/csp/datasets/set02.htm. The 10th SPE Comparative Solution Project was originally proposed as a benchmark for upscaling methods, but the second data set of this benchmark has later become very popular within the academic community as a benchmark for comparing different computational methods. The model is described on a regular cartesian grid, where the model dimensions are $1200 \times 2200 \times 170$ (ft). The top 70 ft (35 layers) represents the Tarbert formation, and the bottom 100 ft (50 layers) represents Upper Ness. The fine scale model size is $60 \times 220 \times 85$ cells (1.122x106 cells).

For simplicity, we only consider a 2-dimensional problem by picking one layer of the oil reservoir of the SPE 10th model. As such, the physical domain D is $[0, 1200] \times [0, 2200]$, and the cartesian mesh has a total of 60×220 cells. For example, the permeability of the 35th layer is plotted in Figure 0.1. The problem of interest is to solve the pressure p and the saturations S_o and S_w by solving the two-phase flow problem in (0.1) and 0.2 with a water injection well and an oil production well shown in Figure 0.1. The injection rate is included in a manner equivalent to boundary conditions, and are left out of the equations in (0.1)-(0.2).

The random permeability field is defined as a Guassian process based on the measurements at the $7 \times 12 = 84$ locations, i.e., the black dots in Figure 0.1. The covariance function of the Gaussian process is

$$\beta(x, y, x', y') := \exp\left[-\frac{(x - x')^2}{x_{\max}^2 L_x^2} - \frac{(y - y')^2}{y_{\max}^2 L_y^2}\right], \tag{0.3}$$

where $x_{\max} = 1200$, $y_{\max} = 2200$. We split the permeability vector $\boldsymbol{K}$ of length 13200 into two groups, denoted by $\boldsymbol{K}_d$ and $\boldsymbol{K}^*$, i.e., $\boldsymbol{K} = (\boldsymbol{K}_d, \boldsymbol{K}^*)$, where $\boldsymbol{K}_d$ are the vector of measurement data and $\boldsymbol{K}_*$ include the unknown permeabilities. In this case, the covariance matrix of $(\boldsymbol{K}_d, \boldsymbol{K}^*)$ can be correspondingly regrouped as

$$Cov = \begin{pmatrix} \boldsymbol{B}_{dd} & \boldsymbol{B}_{*d}^\top \\ \boldsymbol{B}_{*d} & \boldsymbol{B}_{**} \end{pmatrix}.$$

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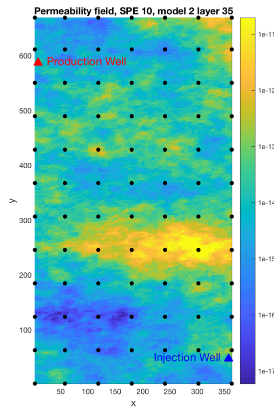

FIG. 0.1. *The 35th layer of the true permeability field of the SPE 10th model 2. The black dots are the locations at which the permeability is measured.*

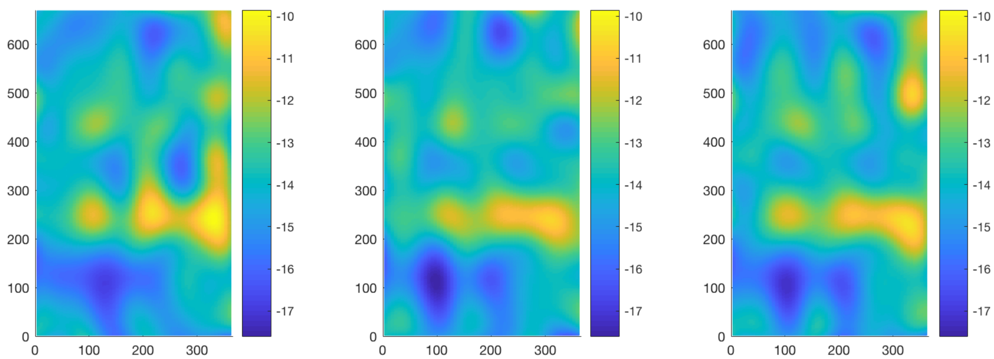

FIG. 0.2. *Three samples of random field conditional on the measurement data at the 84 locations in Figure 0.1.*

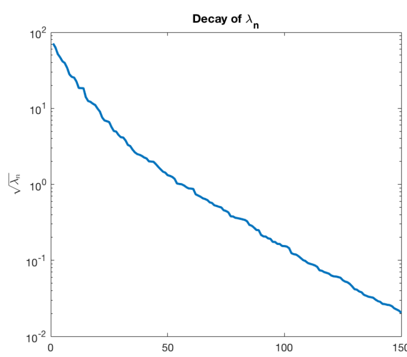

FIG. 0.3. *Decay of* $\sqrt{\lambda_n}$

Then, the conditional covariance matrix for $\boldsymbol{K}_*|\boldsymbol{K}_d$ is

$$\boldsymbol{B}_{**} - \boldsymbol{B}_{*d}\boldsymbol{B}_{dd}^{-1}\boldsymbol{B}_{*d}^{\top}.$$

Three realizations of the Gaussian process with $L_x = L_y = 0.15$ are plotted in Figure 0.2. The Gaussian process can be approximated by the following KL expansion

$$\eta_N(x,\omega) := \sum_{n=1}^{N} \sqrt{\lambda_n}\,\xi_n(x)y_n(\omega), \tag{0.4}$$

where λ_n and ξ_n can be obtained by conducting eigen decomposition of the covariance matrix $\boldsymbol{B}_{**} - \boldsymbol{B}_{*d}\boldsymbol{B}_{dd}^{-1}\boldsymbol{B}_{*d}^{\top}$. For illustration, the decay of the largest 150 eigenvalues are plotted in Figure 0.3.

Black-box simulator. To setup a blackbox simulator for this problem, we utilize the MATLAB Reservoir Simulation Toolbox (MRST) `https://www.sintef.no/projectweb/mrst/`, in which the SPE 10th model is included as a benchmark example. We modified the script for the SPE model in MRST by adding in the random field generation component. In Figure 0.4 we plotted three realizations of the water saturation and the pressure, which are generated using the three random field samples in Figure 0.2.

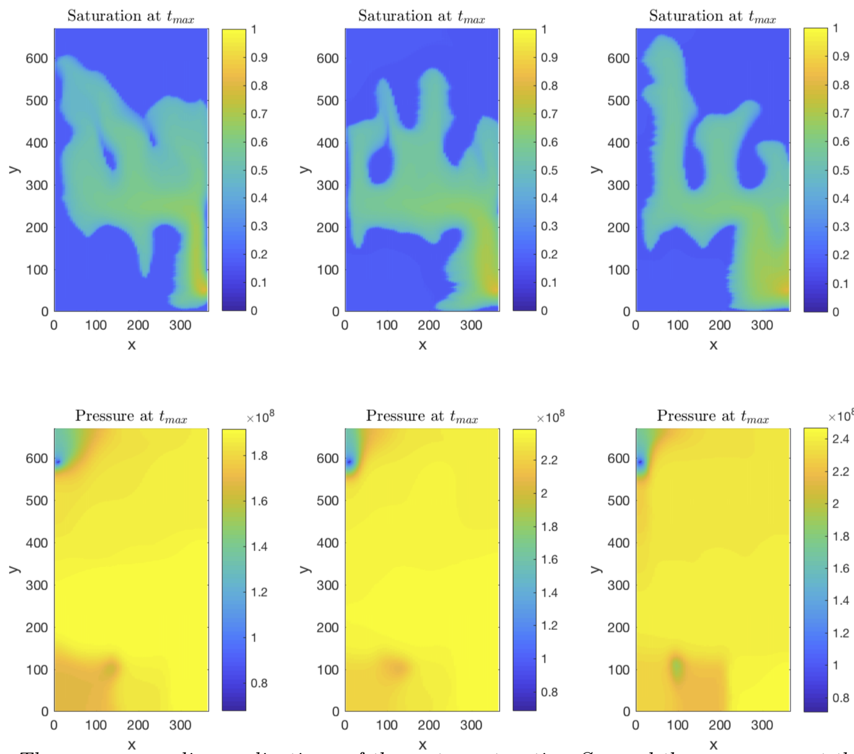

FIG. 0.4. *Three corresponding realizations of the water saturation S_w and the pressure p at the terminal time $t_max = 3000$ days.*