Quantum Annealing as an alternative heuristic for solving combinatorial optimization problems

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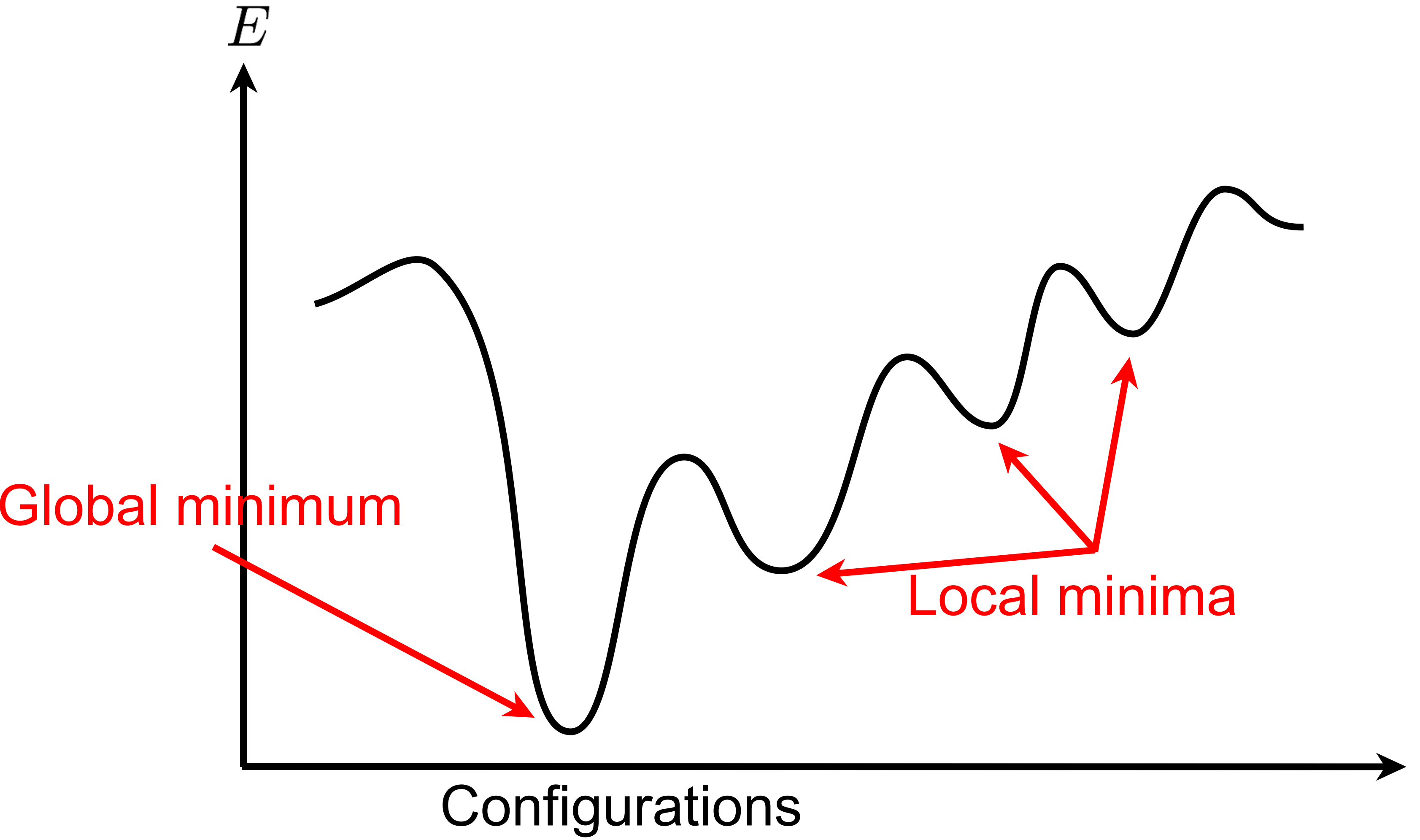
Combinatorial optimization

- Optimize a cost function over a finite (but usually very large) set
- In many cases, best known algorithms scale exponentially with problem size
- Many important practical problems are of this form

Problem	Application
Traveling salesman	Logistics, vehicle routing
Minimum Steiner tree	Circuit layout, network design
Graph coloring	Scheduling, register allocation
MAX-CLIQUE	Social networks, bioinformatics
QUBO	Machine learning, software V&V
Integer Linear Programming	Natural language processing
Sub-graph isomorphism	Cheminformatics, drug discovery
Job shop scheduling	Manufacturing
Motion planning	Robotics
MAX-2SAT	Artificial intelligence

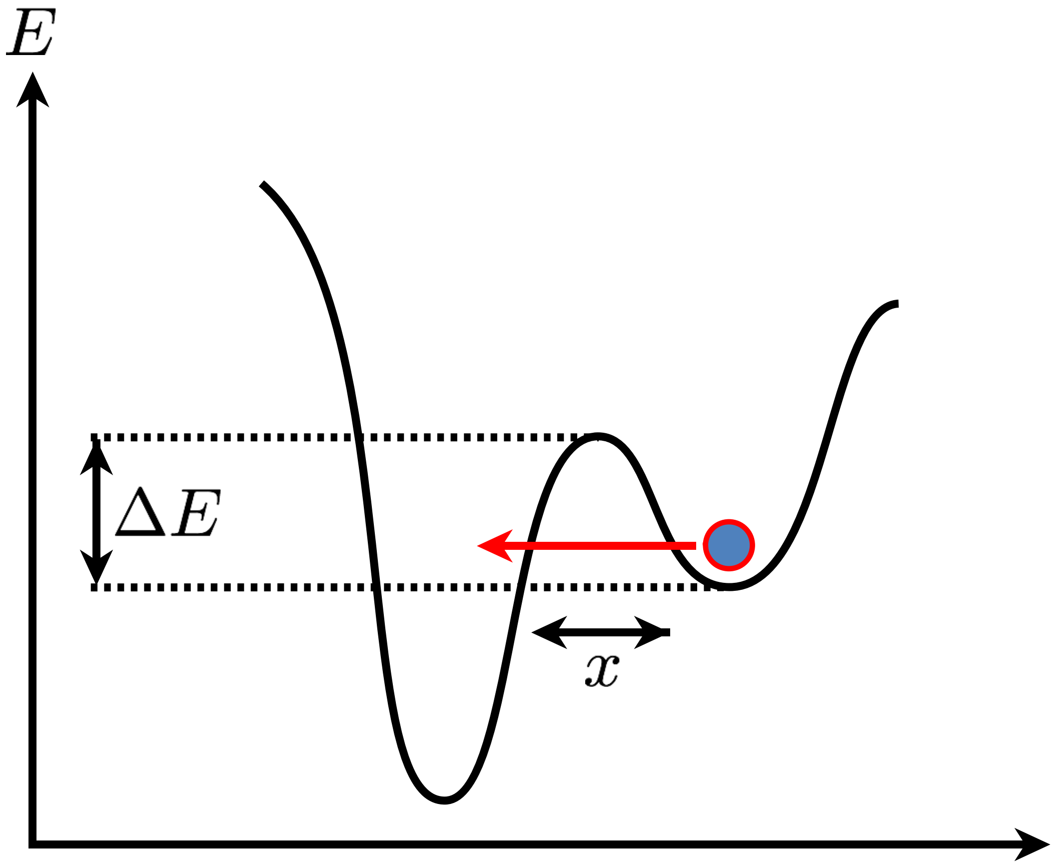
Hardness of CO

- Local minima "trap" many algorithms
- Discrete nature of state space



Quantum Annealing (QA)

Quantum Tunneling



Probability to tunnel through barrier

$$e^{-x\sqrt{\Delta E}}$$

$$x^2\Delta E \begin{cases} \ll 1 & \longrightarrow \text{easy to tunnel} \\ \gg 1 & \longrightarrow \text{hard to tunnel} \end{cases}$$

Quantum Annealing (QA)

Use quantum fluctuations (by tuning a non-commuting field) to escape local minima

Quantum Hamiltonian

$$[\cdot,\cdot] \neq 0$$

$$H(t) = -A(t)\sum_{i=1}^{N}\sigma_i^x + B(t)H_{\text{Ising}}$$

Start at $t = 0$

$$B(0) = 0$$

End at $t = t_f$

$$A(t_f) = 0$$

Initialize in ground state of $H(0)$

End with only $H(t_f)$

$$|\psi(0)\rangle = |\varepsilon_0(0)\rangle \longrightarrow |\psi(t_f)\rangle \overset{?}{\approx} |\varepsilon_0(t_f)\rangle$$

QA as a heuristic for CO

- QA requires less quality of resources than full universal QC
- We have a (candidate) QA implemented
- We need to better understand the usefulness and power of this intermediate step to QC

Some possible uses for QA

- Fast approximates solutions
- Sampling space of solutions (eg., ALLSAT)
- Sampling distributions
- Complement classical algorithms

QA as a heuristic for CO

- <u>Fast approximate solutions</u>: QA may require long times to get exact solution due to errors, but good quality solutions can be generated extremely fast (eg., usec)

- <u>Sampling of solution space</u>: some problems require finding as many solutions as possible (eg., V&V). QA could be designed to sample different regions of the solution space than other classical methods

QA as a heuristic for CO

- Sampling distributions: adjust parameters of QA to approximately sample from different distributions

$$\boxed{\mathrm{QA}\ (\lambda_i)} \longrightarrow P\ (\mathbf{x}\ ;\ \{\lambda_i\})$$

- Complement classical algorithms: combine sampling capabilities and fast generation approximate solutions as subroutines of classical approaches

Summary

- Quantum annealers aim at exploiting quantum mechanics to avoid certain pitfalls of classical optimization methods

- Quantum annealers will likely be easier to develop than a full QC

- Even though they have restricted capabilities, they can still provide a useful complement to classical approaches to combinatorial optimization

- Studying these advantages can also guide us in how to design quantum annealers tailored for different applications (like approximate sampling)