"High-Precision" Quantum Algorithms

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Collaborators: A. Childs (UMD), R. Kothari (MIT)

Related work: D. Berry, A.M. Childs, R. Cleve, R. Kothari, R.D. Somma,
arXiv 1312.1414 (STOC), arXiv 1412.4687 (PRL)

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The problem

Reduce L, the number of gates in quantum circuits

- What to do with a small-to-mid size quantum computer?
- Quantum simulations?
- Solving linear systems of equations?
- Solving differential equations?
- what else?

Observation 1: If we do not have a way to synthesize circuits, L can be ridiculously large [1]

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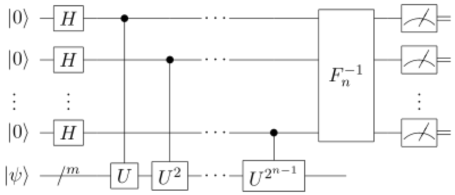

$$\longleftrightarrow$$

$$L \sim \text{poly}(1/\epsilon)$$

- Simulating Hamiltonian dynamics using Trotte-Suzuki decompositions
- Algorithms that use phase estimation as a subroutine

Examples: computation of physical properties, applying inverse of matrices, and more

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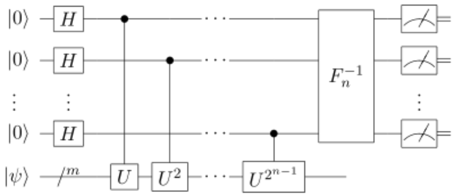

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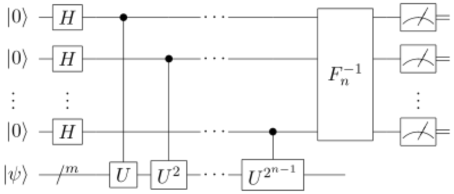

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- Simulating physical systems: real and imaginary time evolutions
- Linear algebra problems: solving linear systems of equations, solving differential equations, etc.

Related work

- Simulating Hamiltonian dynamics [Berry, Childs, Cleve, Kothari, **RS**]
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Useful tools: Linear combinations of unitaries

Goal: prepare $A|\psi\rangle$, $\|A\| \le 1$.

Theorem (1)

Let $A = \sum_{j=0}^{m-1} \beta_j V_j$, where the V_j's are unitary, $\beta_j > 0$, $\sum_{j=0}^{m-1} \beta_j \le 1$. Assume we have access to a unitary $\bar{V} = \sum_{j=0}^{m-1} V_j \otimes |j\rangle\langle j|$. Then, we can simulate A on a quantum computer with one use of $\bar{V}$.

Proof.

- $|\psi\rangle |0\rangle$
- $|\psi\rangle \sum_{j=0}^{m} \sqrt{\beta_j}\, |j\rangle$, with $\beta_m = 1 - \sum_{j=0}^{m-1} \beta_j$
- $\sum_{j=0}^{m} \sqrt{\beta_j}\, V_j |\psi\rangle |j\rangle$
- $\sum_{j=0}^{m-1} \beta_j V_j |\psi\rangle |0\rangle + |\phi^\perp\rangle = A|\psi\rangle |0\rangle + |\phi^\perp\rangle$

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Innovation

Use the above primitive + other methods to build quantum algorithms for various problems, including physics simulation and linear algebra problems, of complexity $\mathrm{polylog}(1/\epsilon)$.

Preliminary studies

- Recently[2] we showed a way to simulate the evolution operator, $A \propto e^{iH}$, using $O(\log(1/\epsilon)/\log\log(1/\epsilon))$ queries.
- Using the Fourier transform, we can decompose e^{-H} in terms of unitaries. This may lead to an algorithm of complexity $\mathrm{polylog}(1/\epsilon)$ for physics simulation.
- There are many decompositions of $A = 1/H$ in terms of unitaries. Can we exploit them to have an algorithm of complexity $\mathrm{polylog}(1/\epsilon)$ for applying inverses?
- Other linear algebra problems?

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Surely, applying operations such as $1/H$ efficiently is important. But...

- In practice, linear systems of equations are not solved by multiplying a vector by $1/H$. A lot of computations can be reused to solve following instances ($H = LU$).
- Conditioning numbers can be very large
- Reading the full answer or encoding the initial vector can still be time consuming

Challenge: Do we have killer apps? (Stephen's talk)

We need more progress

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- Generalize known tools, and introduce new ones, for other applied math problems (e.g., solving differential equations)
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