Moving targets: Quantum circuit synthesis for fault-tolerant gate sets

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Based on joint work with Alex Bocharov, Krysta Svore, and Nathan Wiebe
arXiv:1404.5320 arXiv:1406.2040 arXiv:1409.3552

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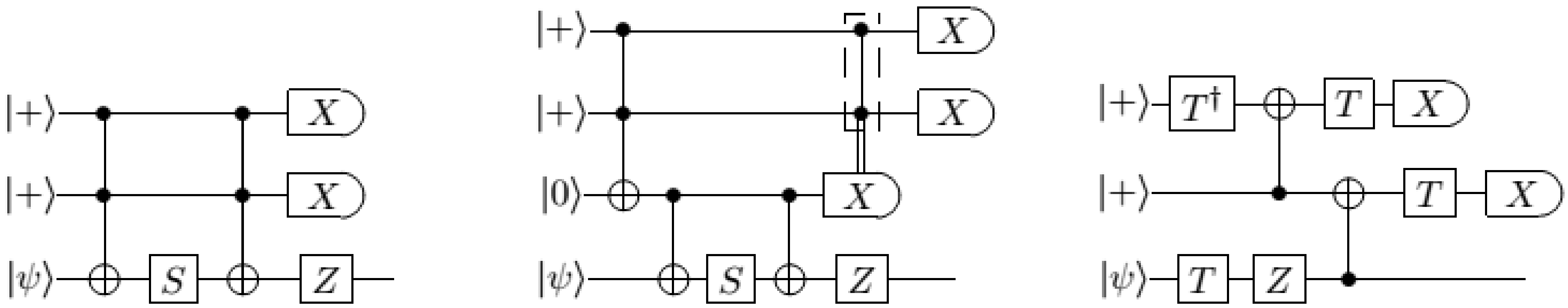
“Repeat-Until-Success” protocols



(a) Exp[T] = 12.8 (b) Exp[T] = 6.4 (c) Exp[T] $<$ 5.26

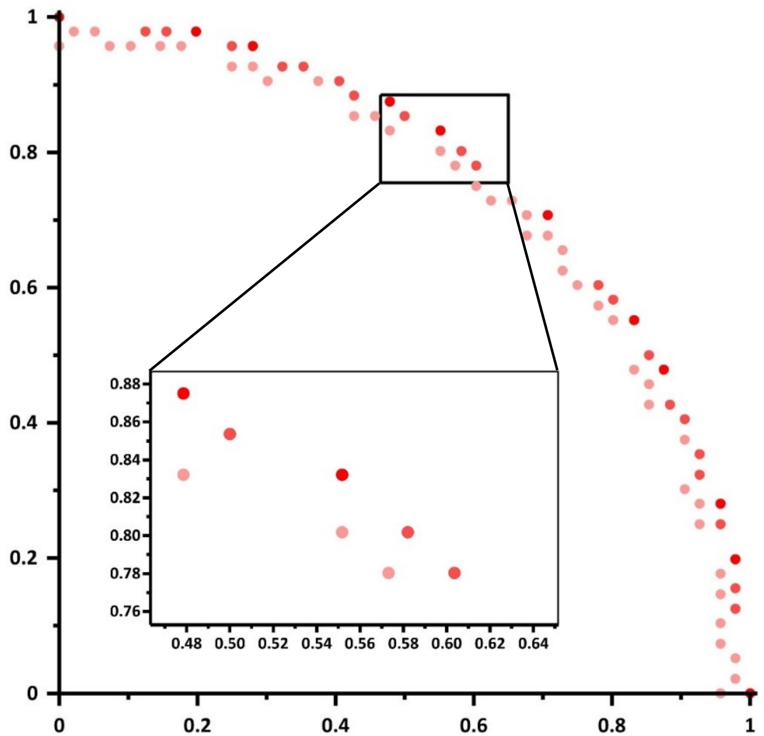
- Implements $V_3 = (I + 2iZ)/\sqrt{5}$ which corresponds to a z-rotation $R_z(\theta)$ with irrational angle θ.
- Probability of success $p_{success} = \frac{5}{8}$
- Failure branches: all lead to identity I_2 [Nielsen/Chuang’00]

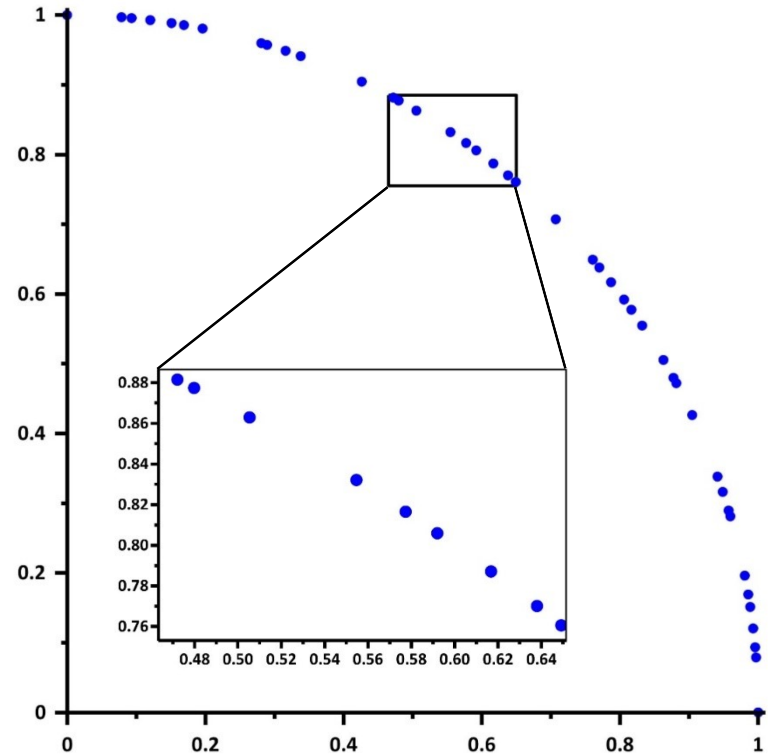
Improved T-count by replacing Toffoli [Jones’12]

Found by enumeration [Paetznick/Svore’13]

RUS synthesis?

Approximations: unitaries versus RUS





a) Complete enumeration $\langle H, T\rangle$ unitaries of T-count $L \leq 8$ (i.e., $\ell \leq 4$). Red/grey denotes if within 0.98 of a z-rotation.

b) Enumeration of RUS protocols with expected T-count $L_{exp} \leq 7.5$, Blue points have $p_{success} \geq 0.8$

Cost advantage: $3.0 \log\left(\frac{1}{\epsilon}\right)$ for unitaries vs $1.15 \log\left(\frac{1}{\epsilon}\right)$ for RUS designs

[arXiv:1403.2975] [arXiv:1404.5320]

Compiling z-rotations: overview

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} z & 0 \\ 0 & z^* \end{bmatrix}$$

$$\downarrow$$

$$u = r\,z/\left(\sqrt{2}\right)^L$$

$$\downarrow$$

$$v = y/\left(\sqrt{2}\right)^L$$

$$\downarrow$$

$$\begin{bmatrix} u & -v^* & 0 & 0 \\ v & u^* & 0 & 0 \\ 0 & 0 & u^* & v^* \\ 0 & 0 & -v & u \end{bmatrix}$$

Rotation angle θ, precision ε

$\downarrow$

Stage 1: Initial approximation of $e^{i\theta}$ (integer relation problem)

$\downarrow$

$z \in \mathbb{Z}[\omega] : z^*/z \sim e^{i\theta}$

$\downarrow$

Stage 2: Randomized search for modifier $r \in \mathbb{Z}[\sqrt{2}]$ with solvable norm equation and high success probability

$\downarrow$

Modification $(rz)^*/(rz) \sim e^{i\theta}$

$\downarrow$

Stage 3: RUS unitary design

$\downarrow$

Two-qubit RUS matrix

$\downarrow$

Stage 4: Synthesis of the RUS circuit

$\downarrow$

RUS circuit for $R_z(\theta)$

Step 1: Integer relation problem: solve it e.g. by using PSLQ or LLL

Step 2: Sample space problem: generate enough modifiers

Step 3: Number theory problem: solve norm equation over $Q(\omega_8)$

Step 4: Implement resulting 2 qubit unitary: exploit special form

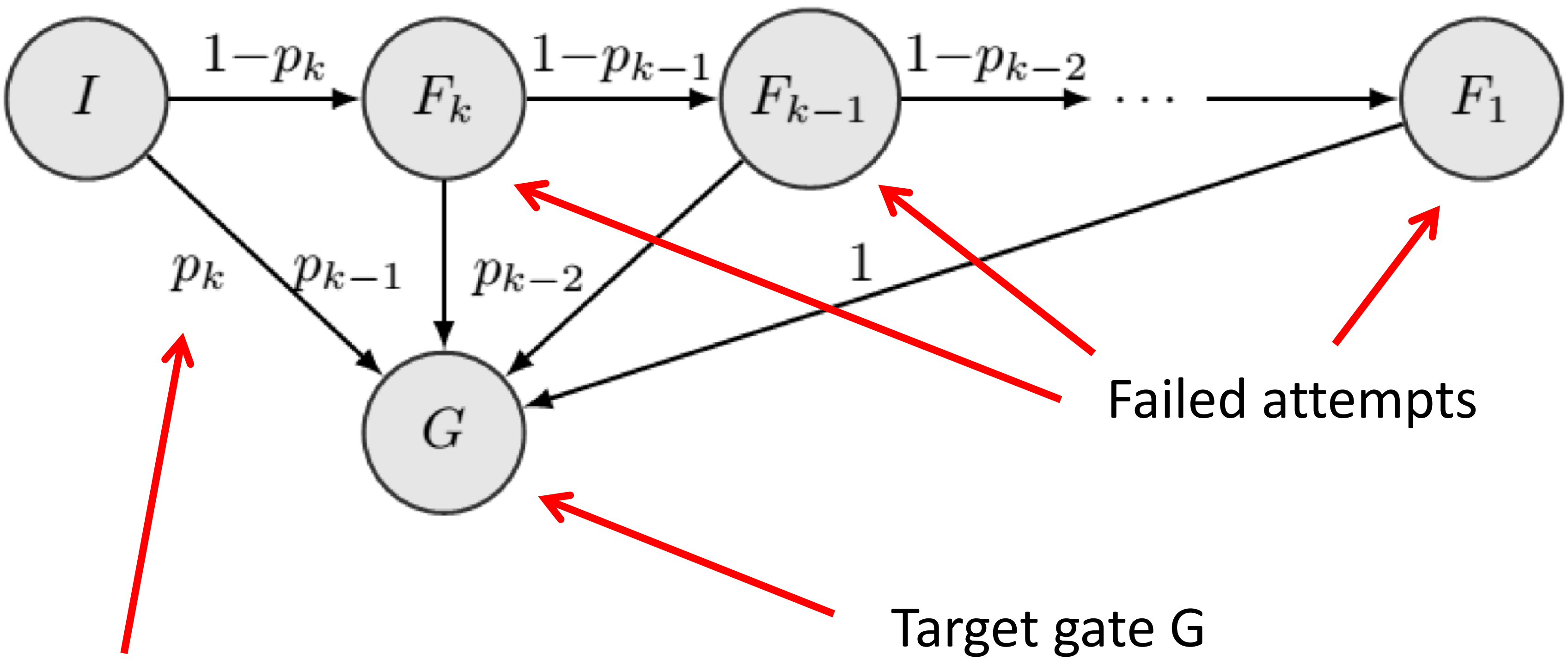
Compilation time: All compiling stages run fast in practice, even for extremely high accuracy (e.g., $\epsilon = 10^{-100}$).

Some open problems

- What are common features of the underlying fault-tolerant gate sets that can be leveraged for the development of an---ideally, systematic---approach toward quantum circuit synthesis?
- How can the synthesis layer inform the fault-tolerance layer about which gate sets are preferable over others?
- How can the trade space between resulting quantum circuit depth (or size) versus number of qubits versus error probability be explored in a meaningful way? Does is make sense to trade compilation time for quality of the resulting circuits?
- Does offline preparation help to bring down the complexity and if so, how can the resources be quantified in a meaningful way?
- How can the problem of synthesizing transformations in the multi-qubit case be addressed?

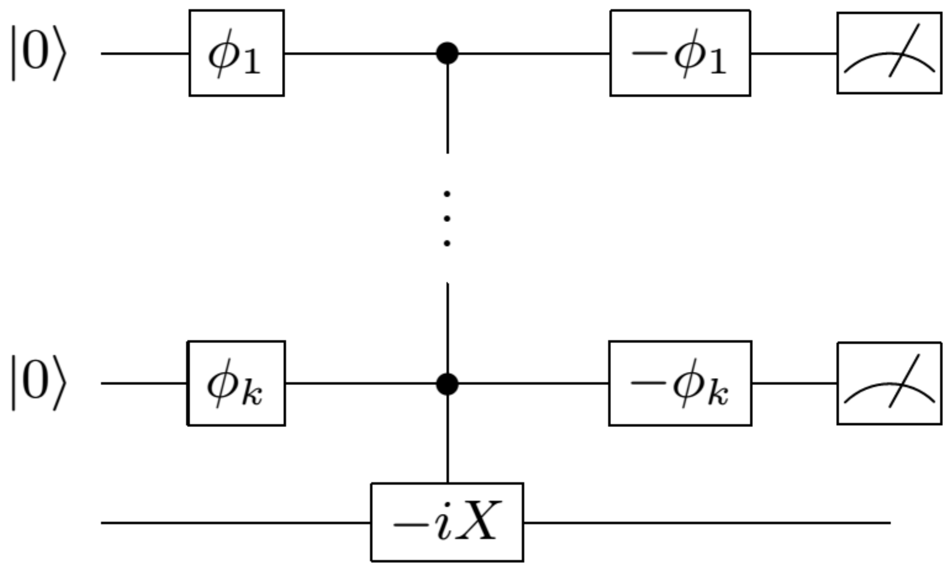
Backup slides

PQF synthesis = finding Markov chains



Success probabilities. Note that in "fallback" designs the last step has P(success)=1

Arithmetic implemented RUS-style



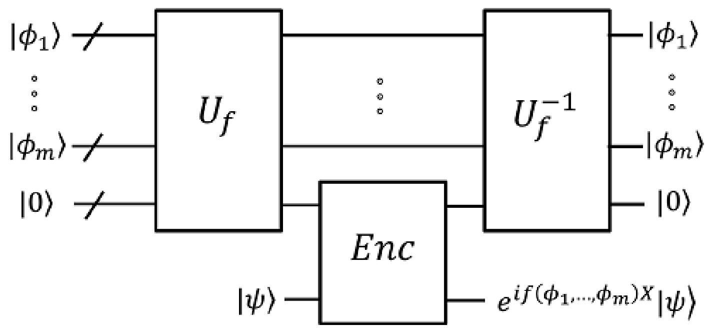
Gearbox circuit with k inputs, where gate ϕ_j denotes $e^{-i\phi_j X}$. Success is achieved if *every* measurement reads 0.

Fact: This defines an RUS circuit. Upon success it implements

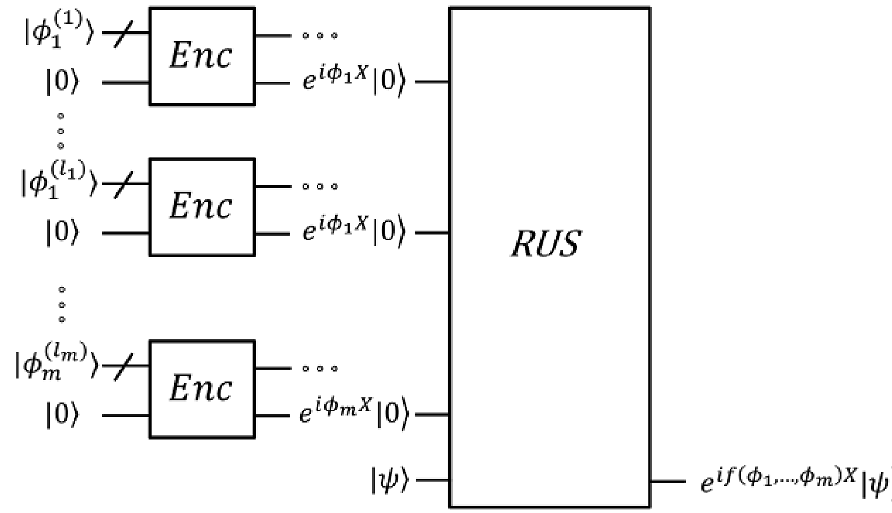
$$\mathrm{GB}\left(e^{-i\phi_1 X}|0\rangle \cdots e^{-i\phi_k X}|0\rangle|\psi\rangle\right) = e^{-i\tan^{-1}(\tan^2(\arcsin(|\sin(\phi_1)|\cdots|\sin(\phi_k)|)))X}|\psi\rangle .$$

Comparison: classical vs quantum

Classical/reversible:



Quantum using RUS synthesis:



Multiplier method	$n=2$ T–count	qubits	$n=4$ T–count	qubits	$n=8$ T–count	qubits	$n=16$ T–count	qubits
Carry-ripple	2.34E+02	4	7.84E+02	8	2.80E+03	16	1.06E+04	32
Table-lookup	3.38E+03	2	3.26E+06	2	3.98E+09	2	1.13E+13	2
M_4	6.11E+01	3	1.97E+03	4	4.64E+04	4	3.00E+07	4
M_6	7.71E+02	4	1.67E+03	4	3.82E+03	4	5.21E+05	5