Advances on Domain Decomposition Algorithms for $H(\text{curl})$ Problems

Domain Decomposition and Parallel Computing

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Abstract

We report on ongoing work joint with Dr. Clark R. Dohrmann of the Sandia National Laboratories, Albuquerque, NM.

We consider a variational problem, which is positive definite and self-adjoint: Find $u \in H_0(\text{curl}; \Omega)$ such that

$$a(u, v)_{\Omega} = (f, v)_{\Omega} \quad \forall v \in H_0(\text{curl}; \Omega),$$

where

$$a(u, v)_{\Omega} := \int_{\Omega} [\alpha \nabla \times u \nabla \times v + \beta u \cdot v] \, dx,$$

$$(f, v)_{\Omega} = \int_{\Omega} f \cdot v \, dx.$$  \hspace{1cm} (1)

We note that $\|u\|_{H(\text{curl}; \Omega)}^2 = a(u, u)$ for $\alpha = \beta = 1$. We use an essentially boundary condition, i.e., the tangential component of $u$ vanishes on $\partial \Omega$. We assume that the coefficients $\alpha(x)$ and $\beta(x)$ are constant in the subdomains into which the given domain $\Omega$ has been decomposed. We approximate this problem using the lowest order Nédélec elements. They are $H(\text{curl})$-conforming with constant tangential components on each edge of the triangulation and with these values common across each edge.

Our domain decomposition algorithms are defined in terms of local solvers on these subdomains or the unions of pairs of subdomains, which share a subdomain edge, in two dimensions, or a subdomain face, in three dimensions, and one or two coarse problems which provide for global transport of information in each iteration.

There are so far three subprojects. In the first, two-dimensional problems were considered for quite general subdomains, which are not necessarily Lipschitz but only uniform. The condition number bound for the preconditioned operators is essentially of the form $C(1 + \log(H/h))^2$, with $C$ a constant independent of the number of subdomains, their diameters, and the dimension of the local subdomain problems and with $H/h$ the maximum number of elements across any subdomain.

For three dimensions, we have considered BDDC algorithms using the same primal constraints as in Toselli’s work in IMA J. Numer. Anal. 2006. In the most interesting variant there are two primal constraints per subdomain edge; the number of such constraints is effectively the dimension of the coarse space of the preconditioner. We have been able to improve Toselli’s result and we can also relax his conditions on the coefficients $\alpha$ and $\beta$. Some of our results are described in a conference paper.

In addition, we have reexamined early, very interesting work by Hu and Zou, SINUM 2003. Their algorithm has two coarse problems which resemble those of classical wire-basket-based domain decomposition algorithms. We have been able to improve their basic bound to $C(1 + \log(H/h))^2$ and also shown how the coarse problems of the algorithm can be implemented effectively.

We note that there has been relatively few papers on domain decomposition theory for $H(\text{curl})$ problems and that those papers are all very technical. An important goal of our project is to develop a new set of simpler tools, which, we believe, will facilitate the development and study of a variety of domain decomposition algorithm.