

***Intrinsic Network Performance in
Transmission Expansion: Maximizing
Volume of Feasible Injections
under project entitled:
Optimization and Control of Electric Power Systems***

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Infrastructure Networks: Common Ground across Domains

- Whether in communication, transportation, or energy, the network is rarely the objective in itself. Network's ability (or inability) to deliver ***indirectly*** determines success in the primary goal of the infrastructure.
- In electricity, dominant costs of production (economic and/or environmental) and benefits of consumption lie at nodes; network determines delivery ***constraints***.
- Network enhancement costs may be non-trivial, but pale next to long-term impact of making new production/consumption possibilities feasible.

Historic Perspective: Electricity Infrastructure Optimization

- Historically, in era of utility as vertically integrated monopoly, long-term planning of generation and transmission was jointly optimized – “Integrated Resource Planning.”
- Restructuring of last two decades has large portions of U.S. operating with competitive wholesale markets for provision of generation.
- In these regions transmission operated by ISO’s as regulated monopoly. Transmission expansion investment on guaranteed rate of return basis.

Critique of Grid Planning Today

- Where wholesale energy markets exist, legally mandated separation of generation planning from transmission planning – wildly different from past.
- Yet approach to transmission planning seems largely a scaled-up version of 25 years ago: **assume** a modest number of scenario for future generation and load, identify transmission expansion to yield lowest long-term production cost for each.
- Seek compromise solution, “ok” for each scenario. If sounds too simplistic with billions \$’s at stake, don’t take my word ... May 19, 2011 New York Times



Critique of Grid Planning Today

The New York Times



May 19, 2011

Testing 8 Different Scenarios for a Future Power Grid

By PETER BEHR of [ClimateWire](#)

Despite competing interests and agendas, members of a utility-sector task force are completing the initial phase of a first-ever attempt to create a long-range transmission plan for the vast Eastern Interconnection power grid that stretches from Saskatchewan to Florida.

The hard part comes after that.

This summer, a series of computer runs will flesh out eight possible scenarios for the future shape of generation and transmission resources in 2020-2030 throughout the interconnection, the synchronized network of high-voltage power lines east of the Rocky Mountains.

Representatives from industry, state regulatory commissions, regional transmission organizations, environmental groups and consumer organizations have created the scenarios to test the impact of a wide range of possible energy policies on the Eastern grid.

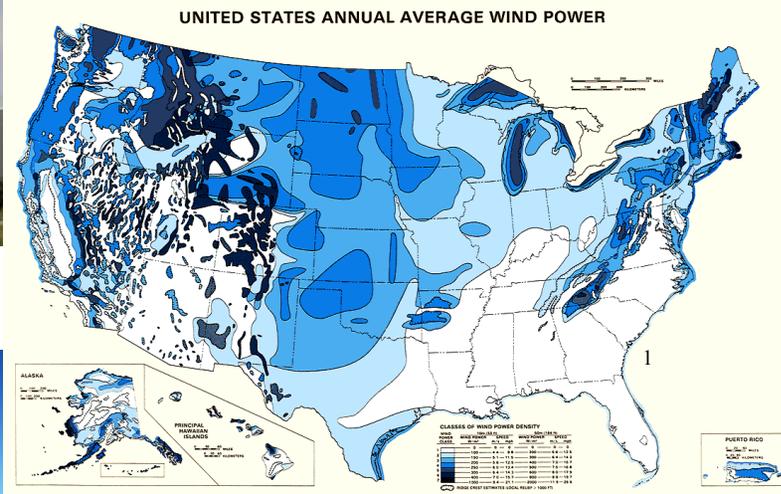
Critique of Grid Planning Today

Slightly oversimplifying..

- Current approach computes very large number of short term (hourly) optimizations of production cost, for long time horizon of interest (say years 2020-30, as per NYT article), over set of selected scenarios, considering possible transmission upgrades, adding amortized capital cost of transmission upgrade.
- As noted, transmission network decisions **primarily** affect constraints on each short term production cost problem, secondarily add capital costs.

Critique of Grid Planning Today

- Range of geographic locations, types, sizes of generation huge: (IMO) eight scenarios don't cut it



Goals for an Alternate Approach

- Seek a measure of transmission performance intrinsic to the network itself, independent of specifics of generation location or characteristics.
- Develop computationally efficient algorithms for optimally sizing and siting transmission expansion, based on this metric.
- Caveat: if current practice weak in its excessive reliance on small number of specific scenarios, method here goes far to other extreme – “scenario free.” A more complete *practical* answer will likely benefit from balance of both.

Specifics of Approach: Feasible Set Volume Maximization

- So, as alternative to approaching transmission expansion decision indirectly, through transmission's constraints on production cost problem, formulate objective directly on transmission network.
- Seek to make network maximally “flexible,” within constraints on number and strength(costs) of transmission links added to base case network.
- In particular, maximize volume of the set of feasible power injections (i.e. range of allowable load and generation powers at all nodes).

Conceptual View

Underlying, Primary Constraints: Set of Allowable Line (Link) Flows. These are a hyper-rectangle

Power Transfer Distribution Factor Map ("PTDF") – widely used computation in power grid

Polytope of Feasible States (node phase angles)

Set of Interest: Polytope of Feasible Bus Power Injections

Linearized Power Flow Map. Here a new link appears as a rank one modification, linear w.r.t. "strength" of the added transmission line

Tricks for Tractability

- Given a network of n nodes, set of possible link additions is order n -choose-2: any two nodes are candidate terminal points for new transmission line.
- Geographic siting constraints will significantly reduce this number, but given practical $n \approx 80,000$, still *VERY* large. Willing to accept some approximations/tricks for computational tractability.

Tricks for Tractability

- Trick #1: employ approximations consistent with widely utilized DC power flow.

Benefits: Reduces dimension of state variables (by assuming secondary set of variables approximately constant); linearizes nonlinear maps.

- Trick #2 (KEY!): Assume any new transmission line added to network is sufficiently high capacity that its flow limit will never become a binding constraint.

Benefit: Set of feasible state variables remain fixed, independent of any line addition!!.



Tricks for Tractability

- Once set of feasible state variables is assumed fixed, focus entirely on linearized power flow map.
- With feasible states polytope **fixed** (and hence of fixed volume), any increase in volume for set of feasible injections is determined by determinant of linearized power flow mapping. Moreover, linearized powerflow map well approximated by matrix with generalized Lapacian structure.
- Problem becomes one of maximizing determinant of a Laplacian, over set of feasible link additions.



Tricks for Tractability

- Even with identification of nicely structured determinant maximization problem, still faced with VERY large number of candidate line additions.
- As bounding method to select tractable number of candidate line additions, seek very low cost method to approximately rank impact of line addition on volume of feasible injection set.
- Overbounding hyper-rectangle for feasible bus injection set. Upper bound on magnitude power at any bus is sum of transmission line strengths incident on bus; i.e., weighted node degree.

Illustration on Example Systems

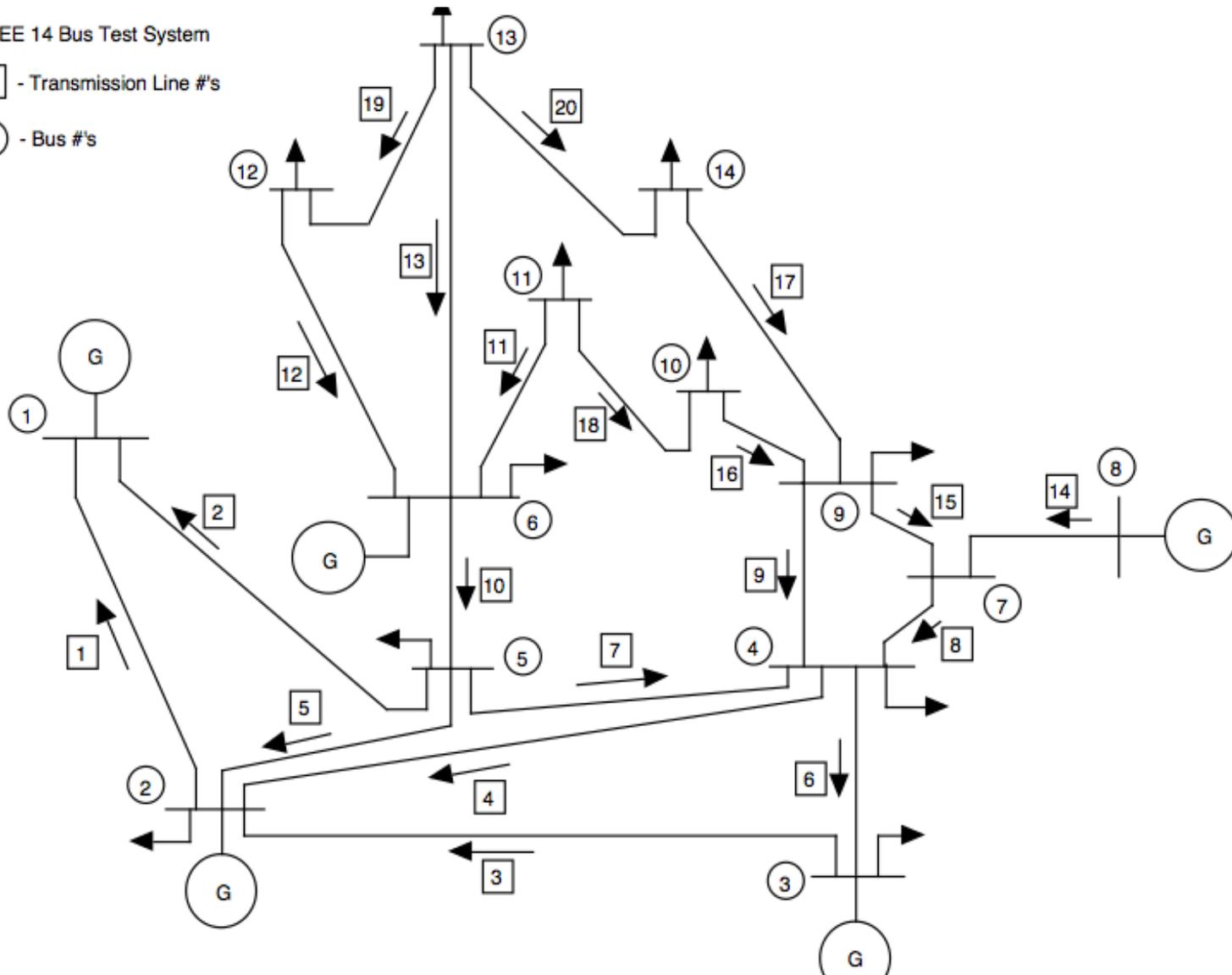
- IEEE 14 bus standard test system: small case, topology of network easily displayed, and exhaustive search over 71 possible transmission line additions easily performed.
- IEEE 118 bus standard test system: “large enough” case to be of interest (though still far shy of 80,000 nodes in U.S. eastern interconnect), but benchmarking against exhaustive search over all possible line additions (6724) still tractable.

Illustration on Example Systems

IEEE 14 Bus Test System

- Transmission Line #'s

- Bus #'s



14 Bus Example System Results

Volume computation for IEEE 14 bus system

Line Placement	Exact Rank	Heuristic Rank	Relative Volume		
			$b_{l+1} = 1\times$	$b_{l+1} = 10\times$	$b_{l+1} = 20\times$
8 to 12	1	3	1.612	7.124	13.247
3 to 12	2	8	1.500	5.997	10.995
8 to 14	3	2	1.495	5.950	10.900
10 to 14	38	11	1.305	4.050	7.100
1 to 4	71	70	1.107	2.071	3.143

118 Bus Example System Results

Volume computation for IEEE 118 bus system

Line Placement	Exact Rank	Heuristic Rank	Relative Volume		
			$b_{l+1} = 1\times$	$b_{l+1} = 10\times$	$b_{l+1} = 20\times$
87 to 117	1	1	5.013	41.135	81.269
21 to 87	2	38	4.608	37.081	73.162
22 to 87	3	15	4.572	36.725	72.450
1 to 87	4	97	4.569	36.689	72.378
28 to 87	5	39	4.523	36.226	71.452
2 to 87	6	56	4.504	36.035	71.070
20 to 87	7	29	4.499	35.992	70.984
10 to 87	8	99	4.496	35.960	70.921
13 to 87	9	17	4.477	35.773	70.546
3 to 87	10	138	4.476	35.761	70.521
2 to 29	3500	2132	2.078	11.783	22.566
65 to 116	6724	6696	1.093	1.959	2.918

Conclusions

- Propose a “scenario free” approach to characterizing value of electric transmission infrastructure upgrades: characterize degree to which they enhance overall flexibility in deliverable power.
- Maximizing generalized volume of feasible power injection set reduced to problem of maximizing determinant in a generalized Laplacian matrix associated with the transmission network.
- Candidate line set grows exponentially with network size – approached through efficient (but certainly heuristic) bounding technique. Excellent performance in computational examples to date.



For more detail

Recent submission to IEEE Transactions on Power Systems, September 2011:

“Transmission Expansion via Maximization of the Volume of Feasible Bus Injections,” F. Thiam and C.L. DeMarco, submission TPWRS-00841-2011.

North American Power Symposium, September 2010:
“Optimal Transmission Expansion via Intrinsic Properties of Power Flow Conditioning,” F. Thiam, C. L. DeMarco