Power Grids: Controlling Cascades and Assessing Vulnerability

Dan Bienstock

Columbia

DOE PI, October 2011

Outline

- **Part I**. The N-K problem in AC power flows
- **Part II**. Cascades

Joint with Sean Harnett (Columbia), Taedong Kim and Steve Wright (Wisconsin).

Supported by DOE DE-SC0002283.

AC Model for Power Grids

Power flows and voltages in a power grid are dictated by elementary physical laws (Kirchoff), along with

- physical attributes (e.g. impedances) of the lines connecting nodes;
- (complex) power supplied at generator nodes
- (complex) power consumed at demand nodes

Leads to a system of quadratic equations in complex variables, namely

- complex voltages $V_i = |V_i|e^{j\theta_i}$ at the nodes;
- complex powers $P_i + jQ_i$ at the nodes.

Half of the variables are given:

- P_i, Q_i at the demand nodes;
- P_i, $|V_i|$ at the generator nodes.

Eliminate P_i and Q_i to obtain a nonlinear system $F(V) = 0$

Background: AC power transmission equaitons

→ Voltage at bus (node) $\mathbf{k} = \mathbf{V_k} = |\mathbf{V_k}|\, \mathbf{e}^{\mathbf{j}\theta_\mathbf{k}}$, where $\mathbf{j} = \sqrt{-1}$

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$$\frac{V_k - V_m}{Z_{km}}$$

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or, current injected by $\mathbf{k} = (V_k - V_m) Y_{km}$, where

$\mathbf{Y_{km}}$ = line admittance = $\mathbf{Z_{km}^{-1}}$

(Π-model, simple version)

General case

$$\begin{bmatrix} \mathbf{I_{km}} \\ \mathbf{I_{mk}} \end{bmatrix} = \mathcal{Y}_{\mathbf{km}} \begin{bmatrix} \mathbf{V_k} \\ \mathbf{V_m} \end{bmatrix}$$

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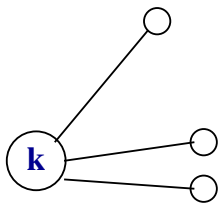

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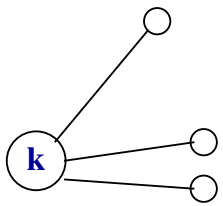

$= \mathbf{U^T}\, \mathbf{Y_k}\, \mathbf{U}$, where $\mathbf{U^T} = (\mathbf{Re(V_1)}, \ldots, \mathbf{Re(V_n)}, \mathbf{Im(V_1)}, \ldots, \mathbf{Im(V_n)})$

Summary:

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Ideally, should meet additional conditions:

$|V_k|$ approximately constant, and high; line phase angle differences small

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We need to solve a system of quadratic equations

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→ The iterative approach works extremely well

→ but only under normal conditions.
→ The rest of the time it's a disaster

Solving the AC Equations

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Lavaei-Low (2010): Semi-definite programming formulation can yield reliable solution (under some assumptions).

Measuring Viability of a Grid

Not all solutions of the AC equations are equally practicable.

- There are often limits on the capacity of transmission lines. Can enforce as constraints on the flows.
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If system not viable, find the smallest total change to the loads that restores feasibility — subject to line limits and voltage magnitudes staying near 1.

Observation: solutions are "sparse"

ℓ_1 Formulation

One possible formulation of the "feasibility restoration" problem:

$$
\begin{aligned}
\min_{V,\sigma,\rho} \; & \sum_{i \in PQ} |P_i^D| \sigma_i \\
\text{s.t. } & F_i^P(V) + |P_i^G| \rho_i = 0, \quad \forall i \in PV, \\
& F_i^P(V) + |P_i^D| \sigma_i = 0, \quad \forall i \in PQ, \\
& F_i^Q(V) + |Q_i^D| \sigma_i = 0, \quad \forall i \in PQ, \\
& V_{\min} \leq |V_i| \leq V_{\max}, \quad \forall i \in PV \cup PQ, \\
& 0 \leq \sigma_i \leq 1, \quad \forall i \in PQ. \\
& \rho_i \text{ free}, \quad \forall i \in PV,
\end{aligned}
$$

where σ_i denote (relative) changes to the loads, ρ_i denotes changes to generation capacity, etc.

Vulnerability Analysis – "N-K" problem

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Problem: Maximize the degradation of the network $\mathcal{G}(\gamma)$ when impedances are changed according to $\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_n)$.

$$\max \mathcal{G}(\gamma) \ \text{ s.t. } \ e^T \gamma \leq B,$$

where $e = (1, 1, \ldots, 1)^T$ and B denotes the "impedance increase" budget.

"Maximize the damage for a given budget of impedance increases."

Large γ_i = critical lines.

Defining $\mathcal{G}$

Possible definitions of degradation measure $\mathcal{G}(\gamma)$:

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In either case, get a two-level optimization problem:

- **Upper level:** variables γ, objective $\mathcal{G}$;
- **Lower level:** solving AC power flow equations (or ℓ_1 extension) for given γ.

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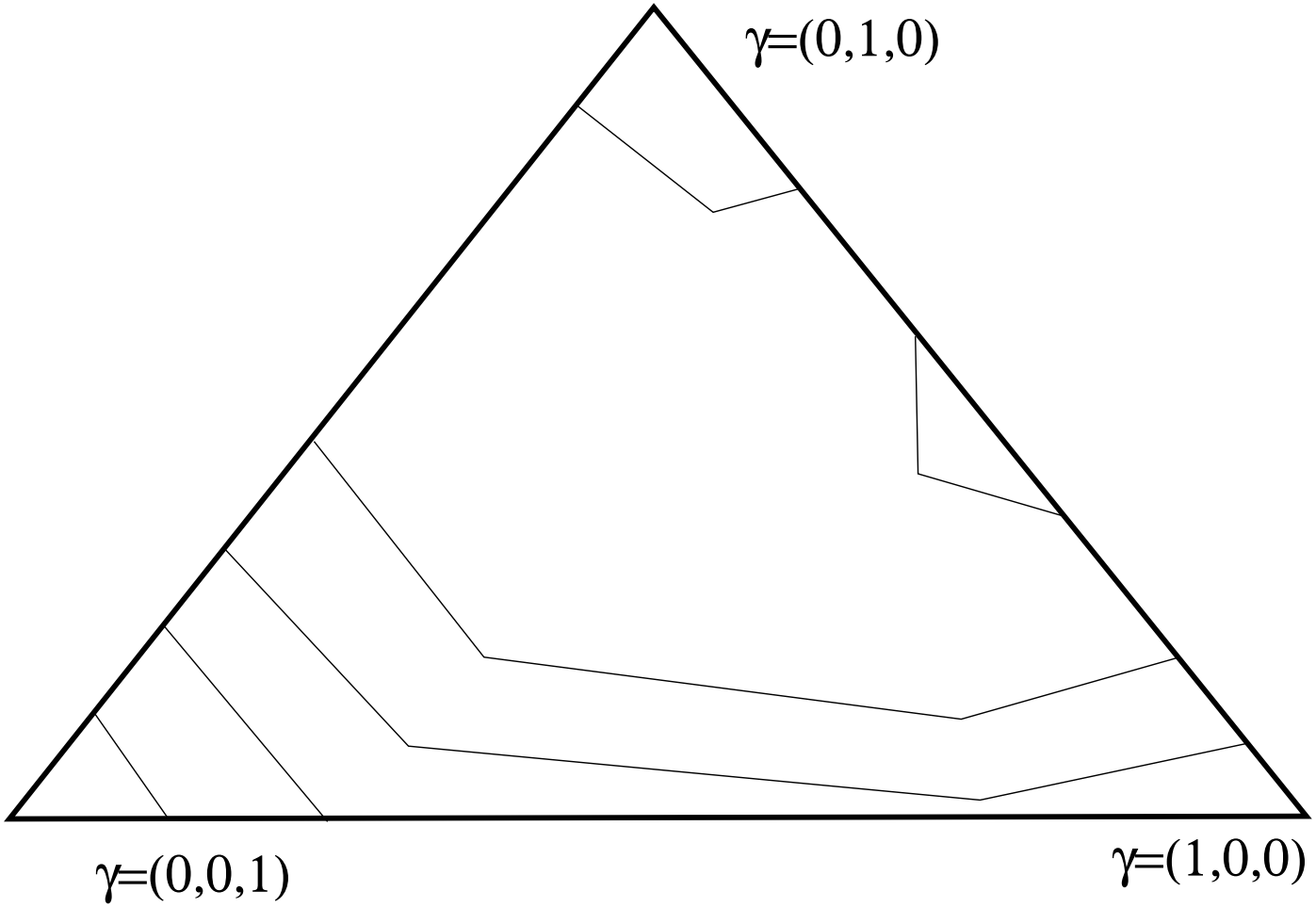
Objective $\mathcal{G}$ usually non-concave, possibly nonsmooth.

- Frank-Wolfe method with estimated gradient (smooth objectives);
- Randomized search algorithm for nondifferentiable optimization (described below).

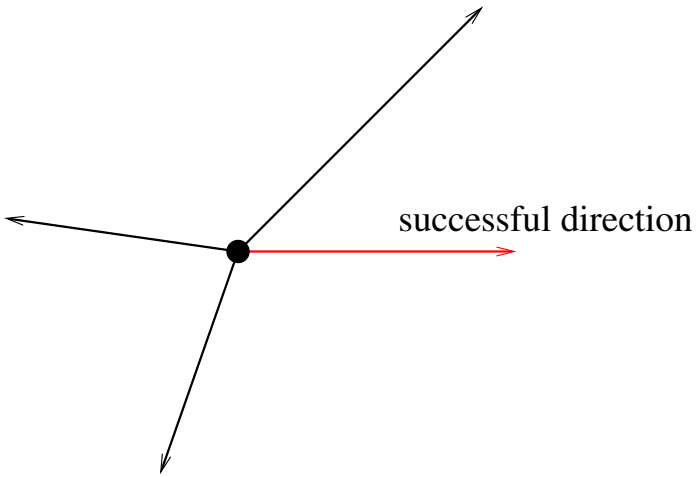
Possible Contours of $\mathcal{G}(\gamma)$

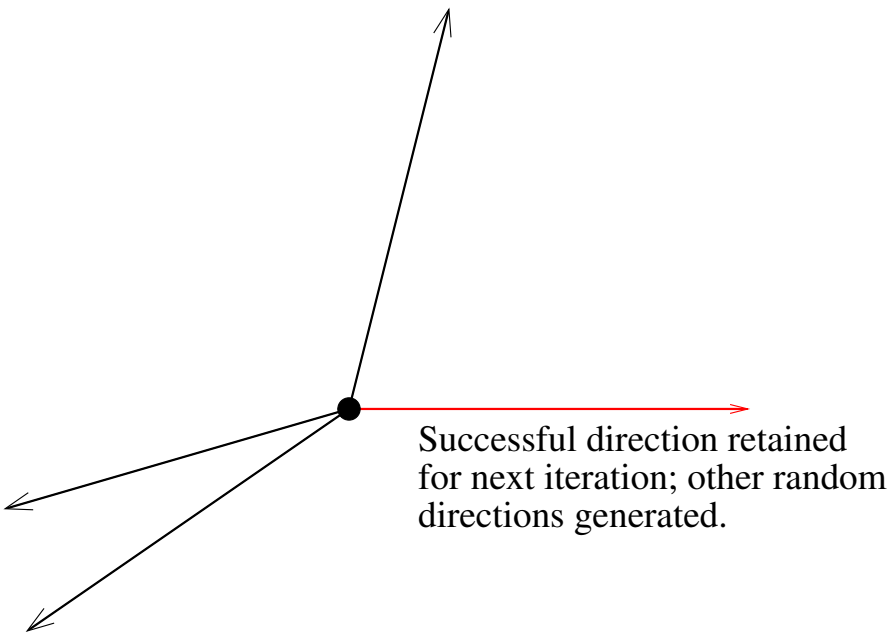


Possible countour map for G in case n=3.
Multiple local maxima; global max is at (0,0,1)

Randomized Search

- Take latest successful search direction and calculate some random new directions (10, say);
- Evaluate function at step length α along each direction;
- Take step with the best decrease.
- If all steps increase f, shrink α and try again with a new set of random directions.





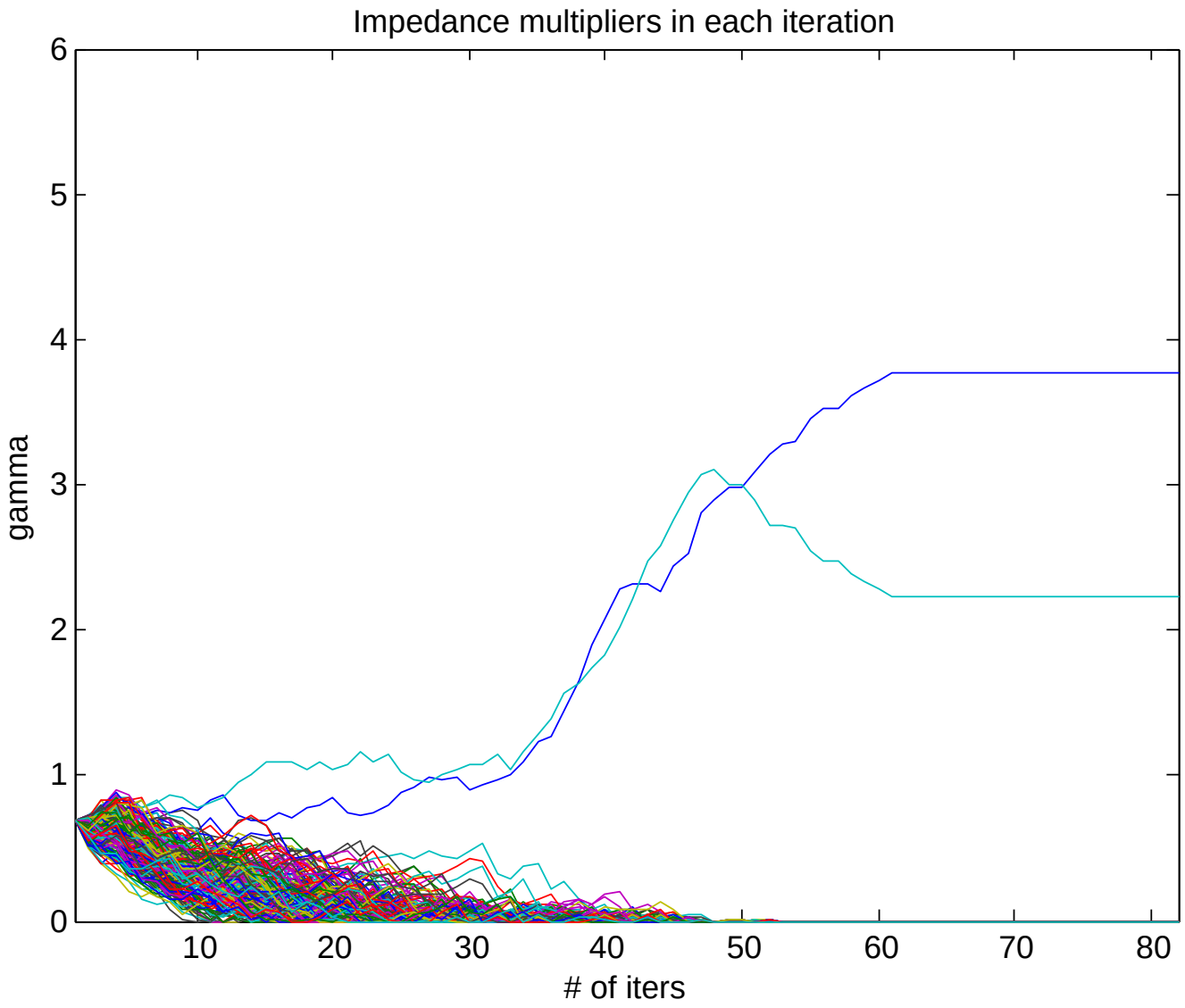
Run 10 times on the IEEE 118 Bus Case

10 random search directions at each step; budget $B = 6$; uses a continuation heuristic (start with large B and decrease to 6).

Run	Number of Iterations	Objective $\mathcal{G}(\gamma)$	Lines with non-zero γ_i			
			71	72	74	82
1	77	1.3309e-01	3.7685	0	2.2315	0
2	109	1.3464e-01	3.1652	0	2.7205	0.1143
3	152	1.1230e-01	2.0779	0.6844	3.2257	0.0120
4	147	1.2033e-01	2.4403	0	3.5597	0
5	71	1.3309e-01	3.7637	0	2.2361	0
6	120	1.2197e-01	2.4565	0.2271	3.3163	0
7	88	1.3309e-01	3.7685	0	2.2315	0
8	109	1.2133e-01	2.3508	0.2450	3.4043	0
9	96	1.3309e-01	3.7683	0	2.2317	0
10	82	1.3309e-01	3.7683	0	2.2317	0

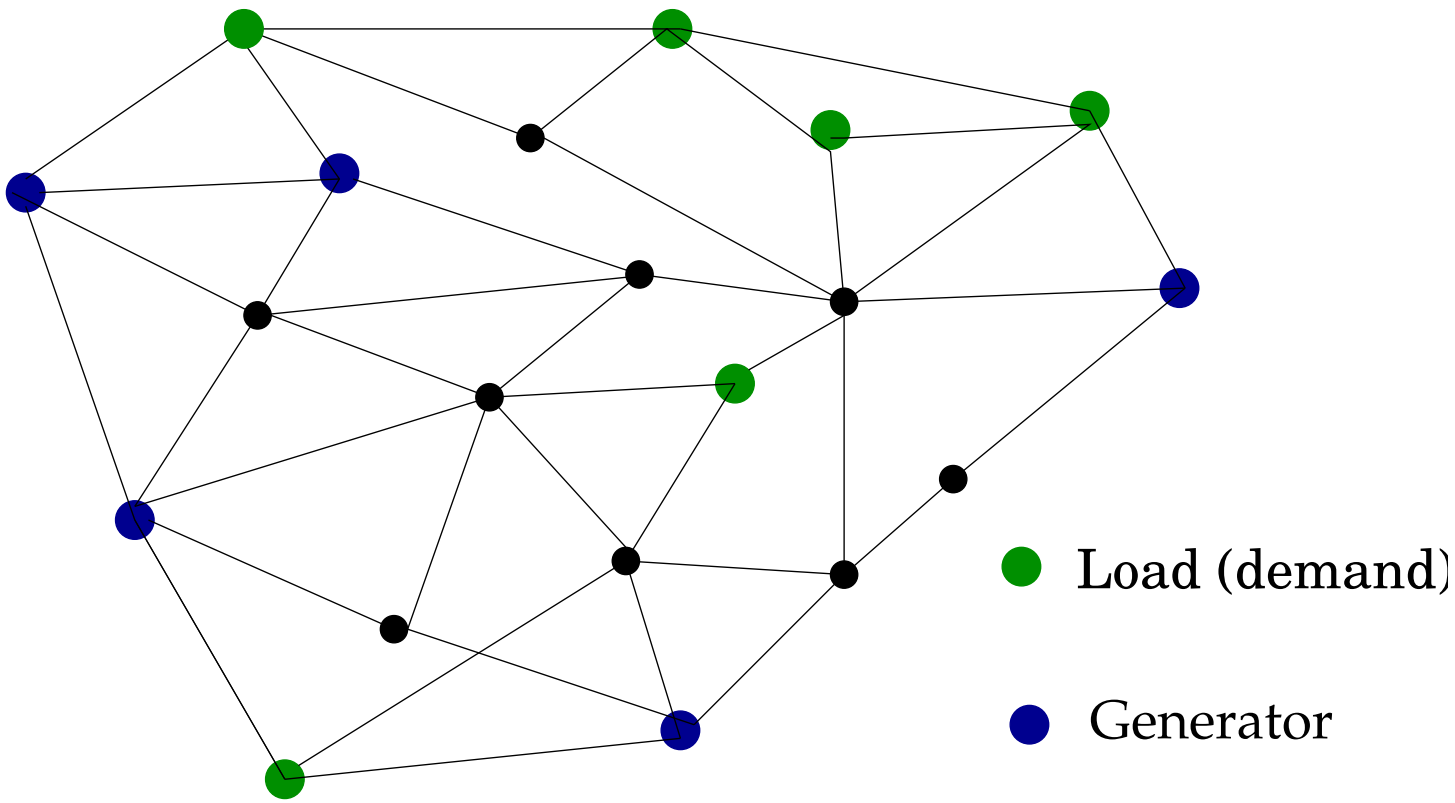
Final solutions similar, but not the same, because of randomness in search.

γ vs iteration count on Run #10



Part II. Cascades

Cascade cartoon - Dobson et al



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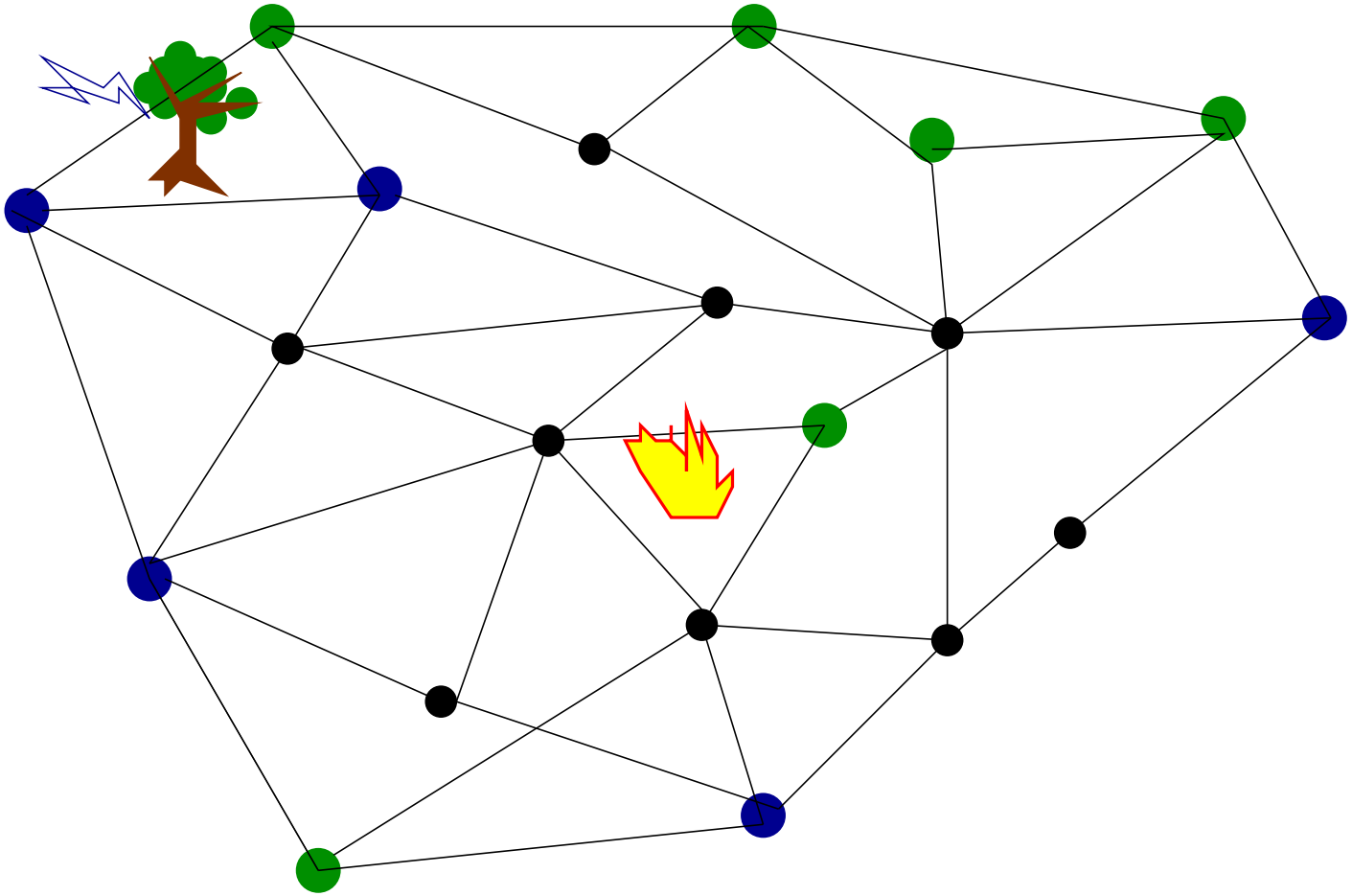

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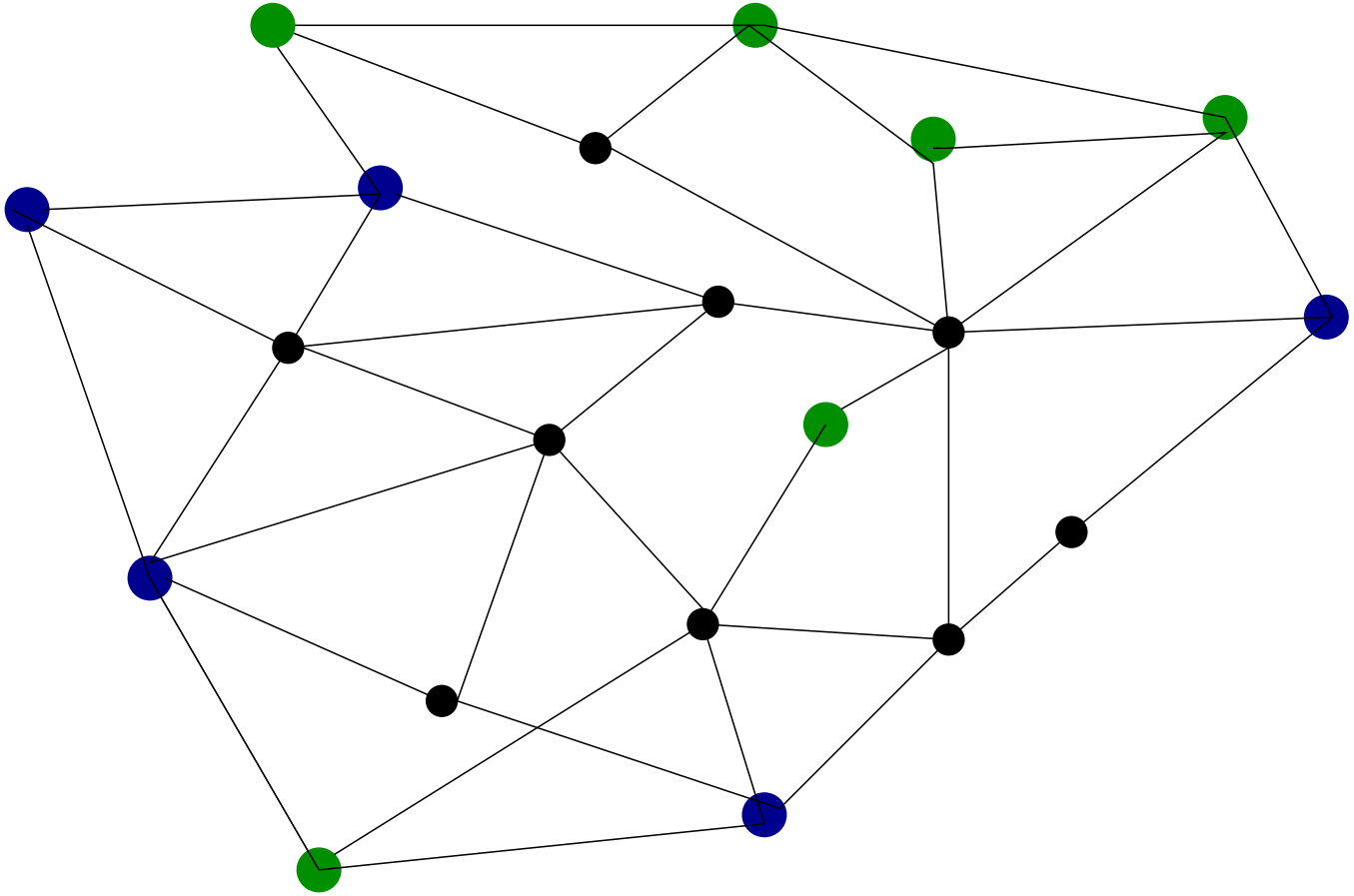

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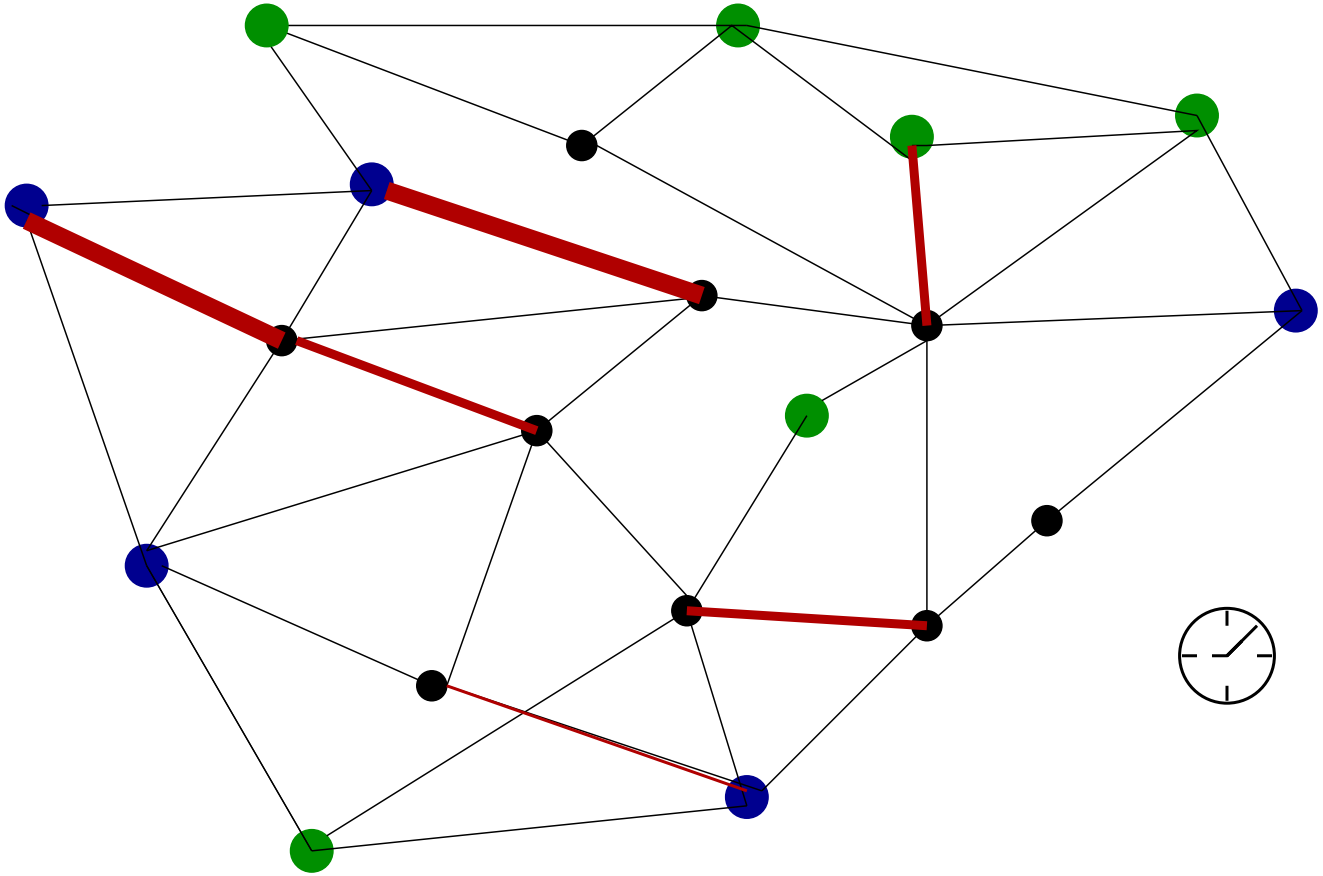

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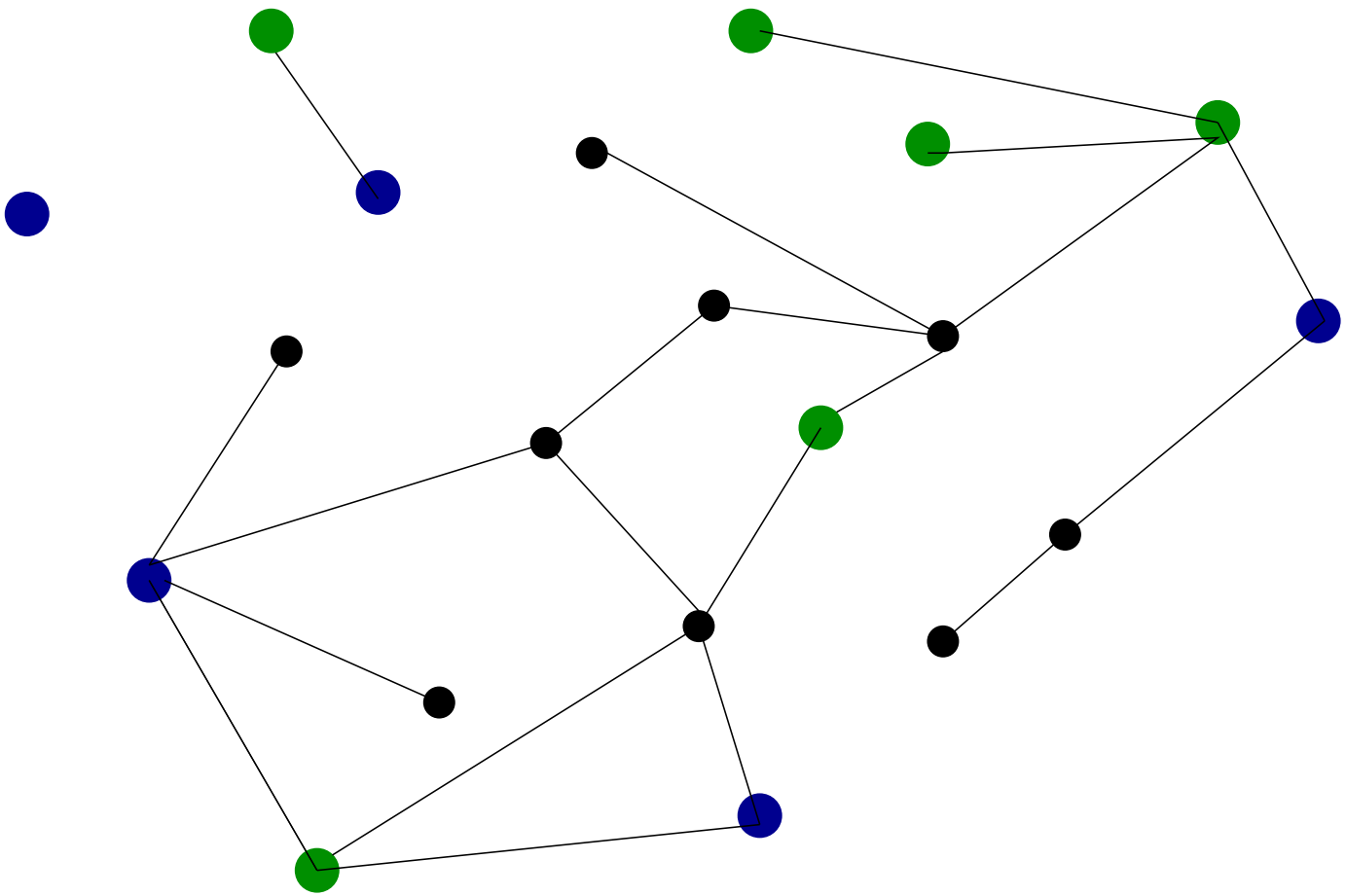

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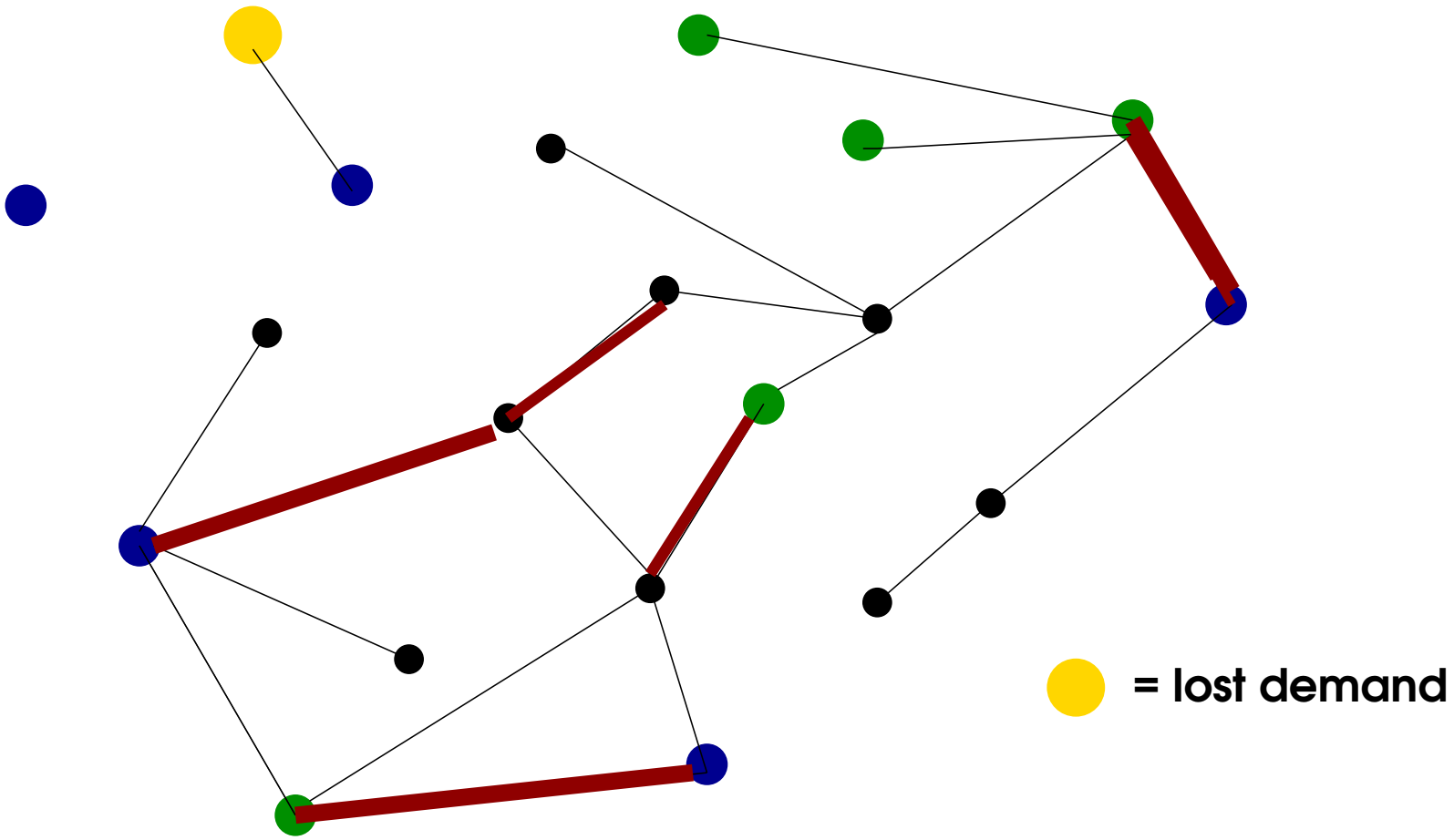

Controlled cascades + detailed cascade model

→ Initial outage event takes place.

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1. Reconfigure demands and generator output levels.

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At round $\mathbf{R}$, reduce demands so as to remove any line overloads.

Robust control

- Partial information
- Noisy or incorrect information
- Delays

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- Let $\boldsymbol{\kappa}^{\boldsymbol{\delta}}$ = max **overload** of any line within radius $\boldsymbol{\delta}$ of $\mathbf{v}$
- If $\boldsymbol{\kappa}^{\boldsymbol{\delta}} > \mathbf{c_v^r}$, demand at $\mathbf{v}$ reduced (scaled) by a factor

$$\min\left\{\mathbf{1}, \left\{\mathbf{b_v^r} + \mathbf{s_v^r}\left(\mathbf{c_v^r} - \boldsymbol{\kappa}^{\boldsymbol{\delta}}\right)\right\}^{+}\right\}.$$

Adaptive affine controls

For each demand bus n $\mathbf{v}$, and round $\mathbf{r}$, control triple $\mathbf{c_v^r}$, $\mathbf{b_v^r}$, $\mathbf{s_v^r}$

$\longrightarrow$ Parameterized by integers $\mathbf{r > 0}$ and $\boldsymbol{\delta} > \mathbf{0}$.

At round $\mathbf{r}$,

- Let $\boldsymbol{\kappa}^{\boldsymbol{\delta}}$ = max **overload** of any line within radius $\boldsymbol{\delta}$ of $\mathbf{v}$
- If $\boldsymbol{\kappa}^{\boldsymbol{\delta}} > \mathbf{c_v^r}$, demand at $\mathbf{v}$ reduced (scaled) by a factor

$$\min\left\{1, \left\{b_v^r + s_v^r\left(c_v^r - \kappa^\delta\right)\right\}^+\right\}.$$

Example: $(1, 1, s)$ control; scale = $\min\left\{1, \{1 + s\,(1 - \kappa)\}^+\right\}$.

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Problem: choose control so as to maximize demand at end of round $\mathbf{R}$.

Methodology

- General approach: simulation-based optimization
- Better fundamentals: stochastic gradient algorithm applied to (randomly) smoothed version of the problem
- Parallel implementation on 40 i7 cores using Unix sockets
- Each gradient computation takes a few seconds (per core) on real grids with tens of thousands of lines

Sept 2011 San Diego blackout

Joint work with A. Bernstein, D. Hay, G. Zussman, M. Uzunoglu (EE Dept. Columbia)

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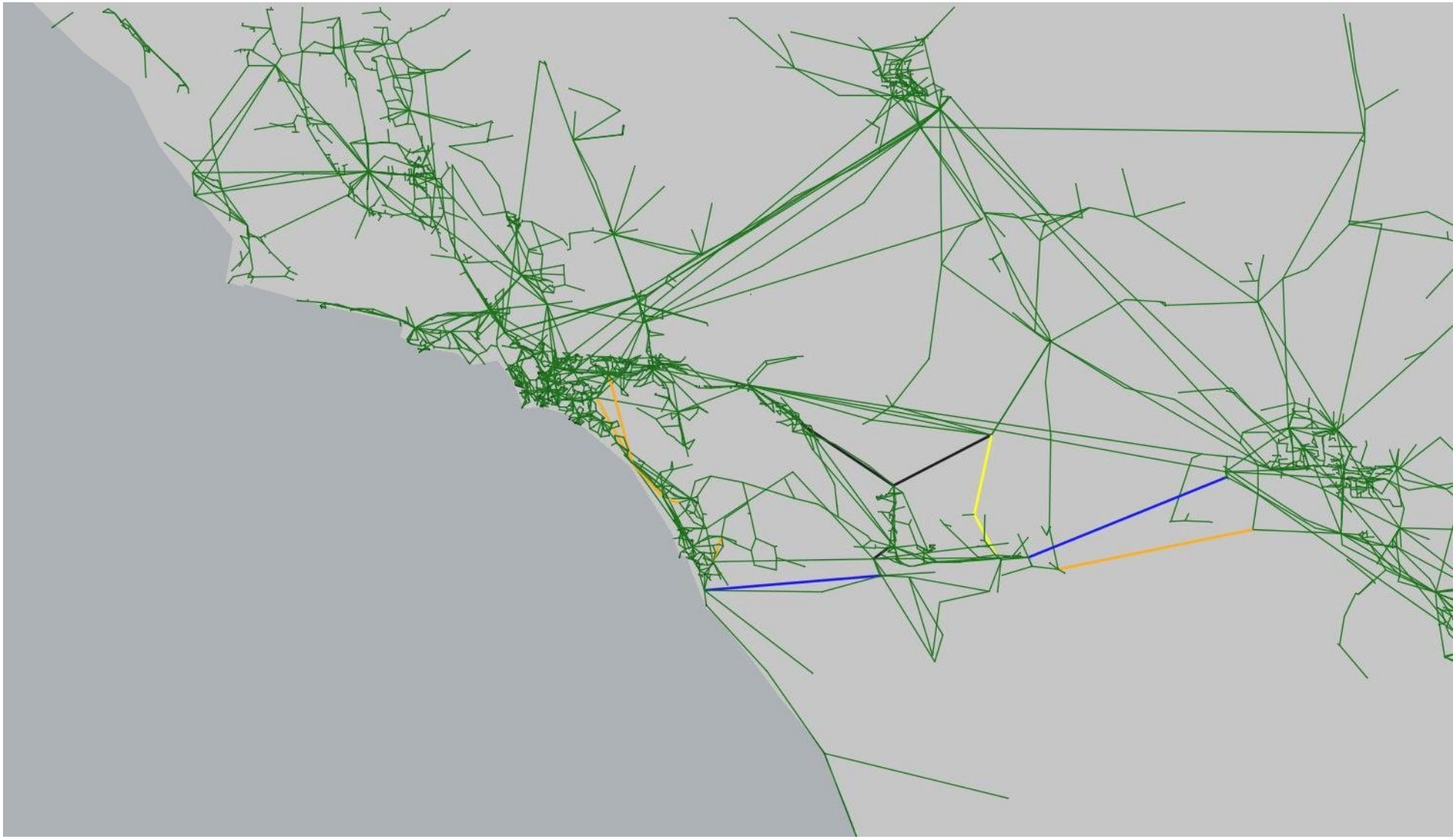
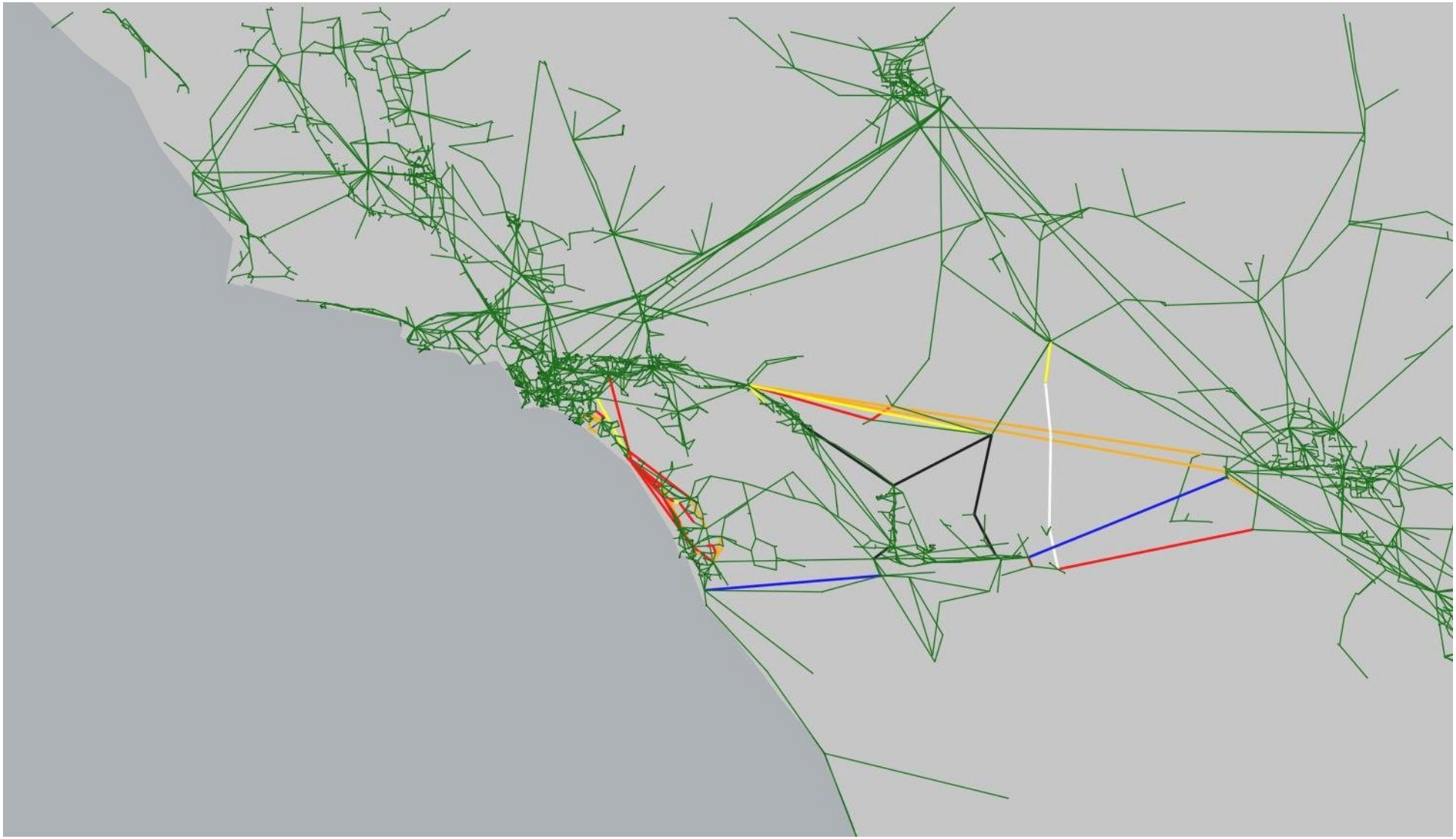
- Data should be made available to **all** researchers

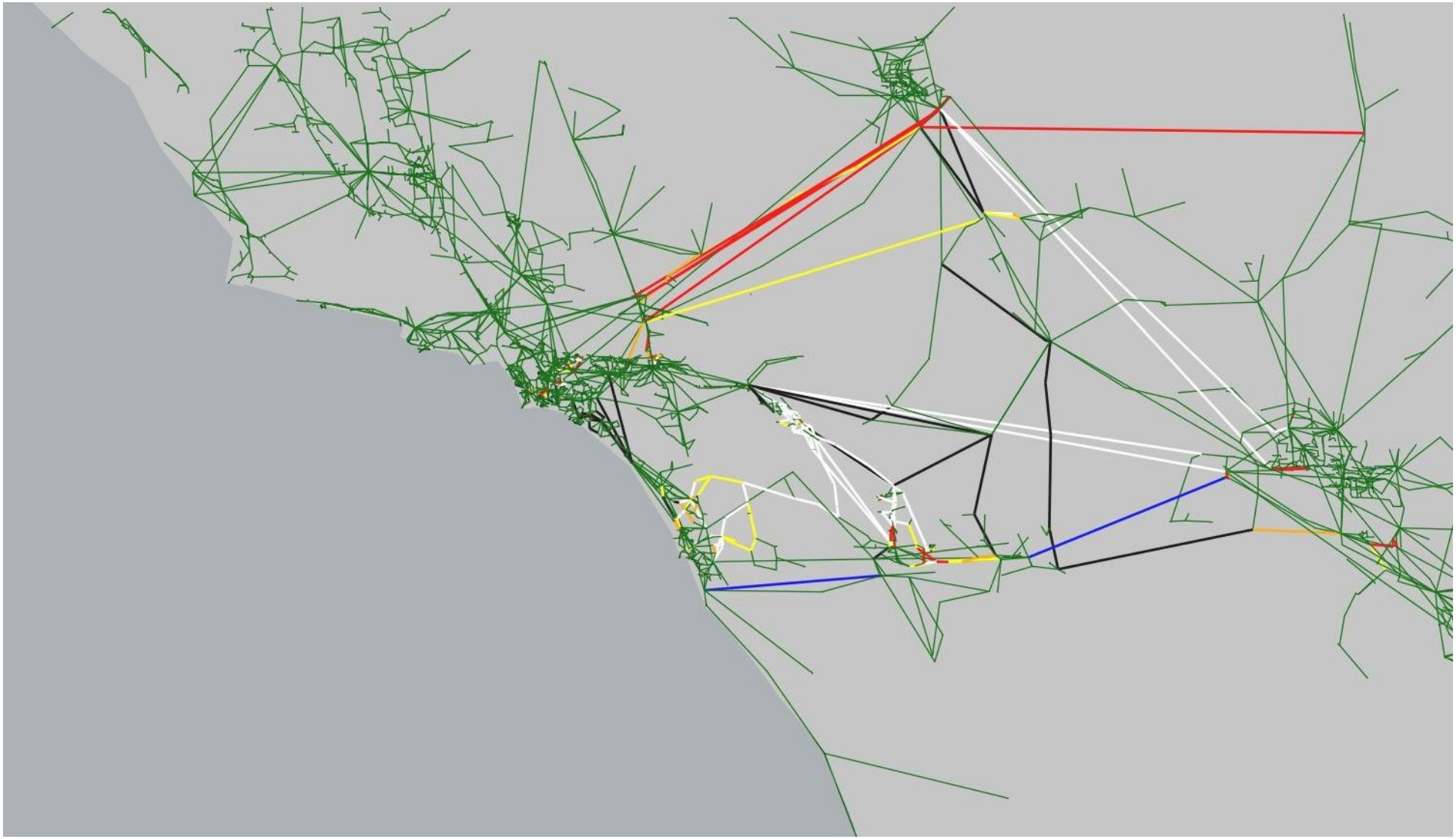
Beginning



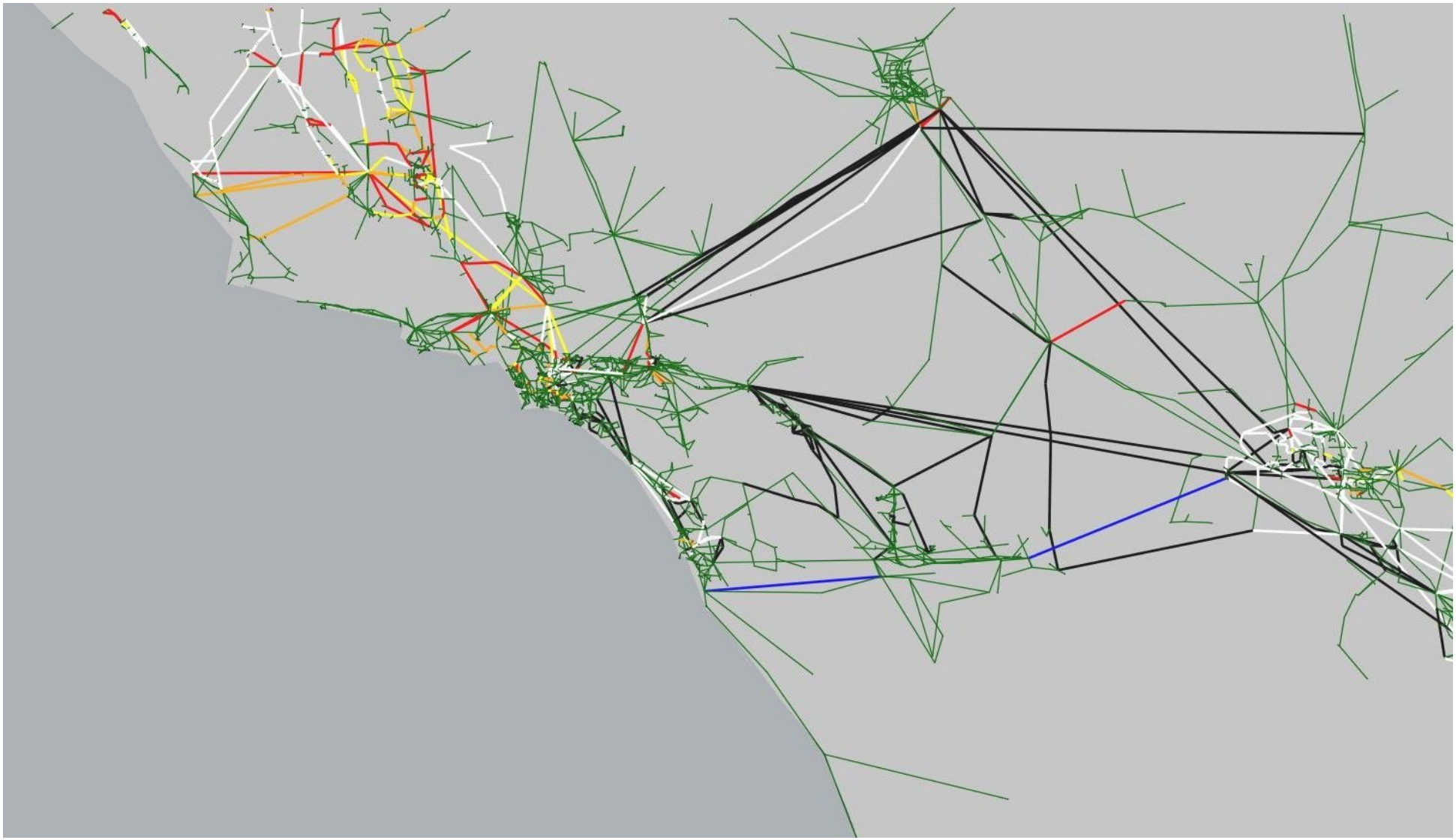
More



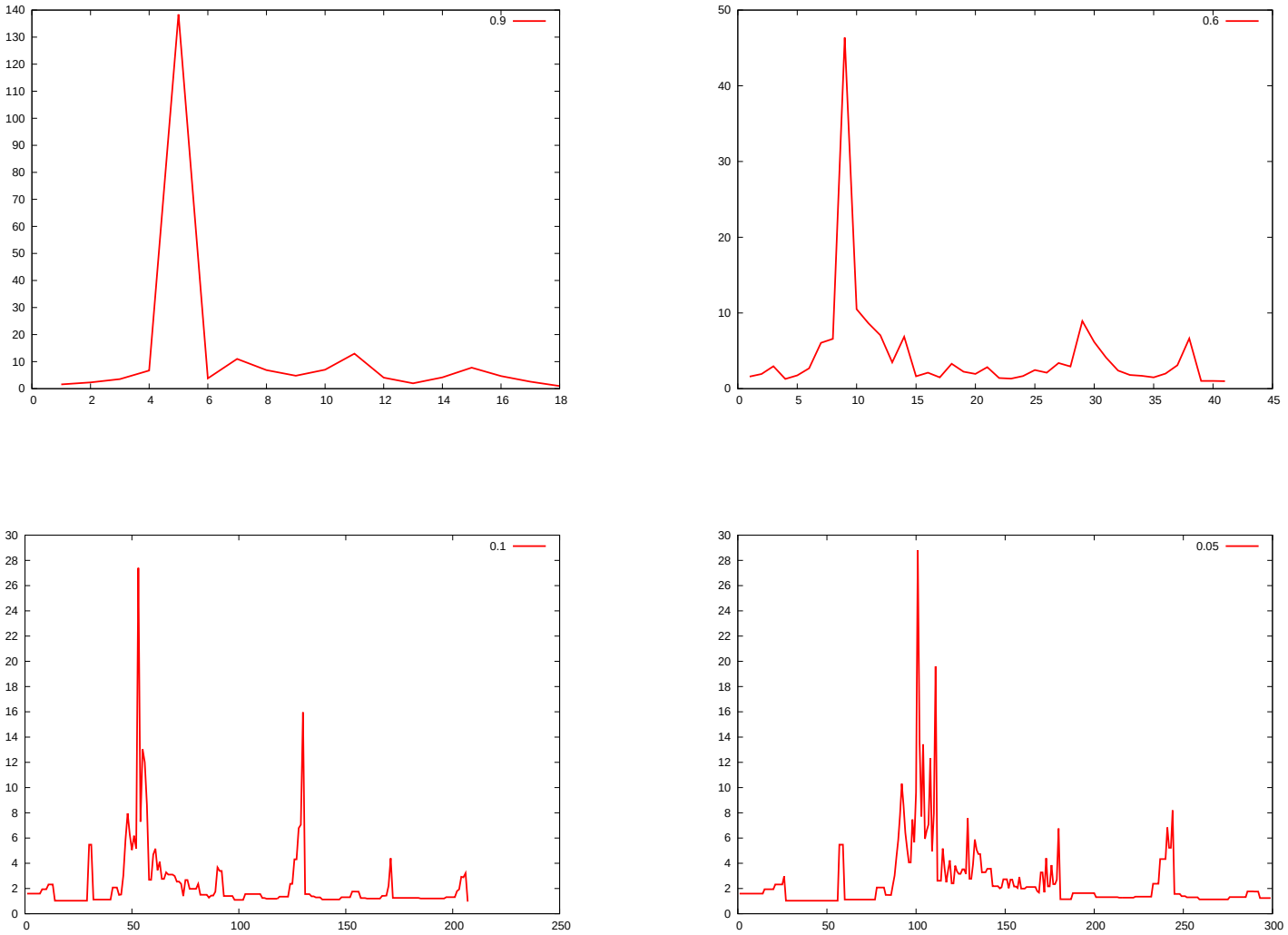
More



More



Max line overload during cascade with increasing resolution



"Ideal" control

Stop cascade by shedding a minimum amount of load

$\longrightarrow$ Remove all line overloads
$\longrightarrow$ System limitations (e.g. generator "ramp up/down" speeds)

(solve an optimization problem)

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But: very nonrobust and requires much coordination

Near optimal scaling control

Simple, robust

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No-control approach:

- yield = **61.47%**,
- $>$ **300** rounds, $>$ **800** outaged lines