A fourth order accurate finite difference scheme for the elastic wave equation in second order formulation[1]

Serpentine: Finite Difference Methods for Wave Propagation in Second Order Formulation

Björn Sjögreen and N. Anders Petersson
Center for Applied Scientific Computing
Lawrence Livermore National Laboratory
P.O.Box 808, L-422
Livermore, CA 94551

Abstract

Immense computational powers are required to simulate wave propagation due to seismic events. For example, to calculate motions relevant for designing buildings and structures, frequencies between 0-10 Hz need to be captured up to 100 km away from the epicenter. Such computations are still far from what can be accomplished, even with today's most powerful supercomputers. There is, therefore, a considerable interest in further increasing the resolution of seismic computations. There are two ways to increase the resolution: a) buying a bigger computer to increase the number of grid points or b) finding a more efficient numerical method. Optimally, a) and b) should be used together. In this presentation we will address b) by developing a fourth order accurate, energy conserving, finite difference method for the elastic wave equation. This research builds on our previous work in seismic wave simulations, which has been based on a second order accurate, also energy conserving, method.

We develop the new fourth order accurate method for the elastic wave equation in second order formulation. Fourth order accuracy holds in both space and time. Discrete energy norm estimates are developed to prove that the method is stable. The stability holds for arbitrary heterogeneous material properties, on multi-dimensional spatial domains with Dirichlet or traction (free surface) boundary conditions.

The key ingredient of the method is a boundary modified fourth order accurate discretization of the second derivative with a variable coefficient, $(\mu(x)u_x)_x$. This discretization satisfies a summation by parts identity that guarantees stability of the scheme. The boundary conditions are enforced through ghost points, thereby avoiding projections or penalty terms, which often are used with previous summation by parts operators. The temporal discretization is obtained by an explicit modified equation technique.

Numerical examples with free surface boundary conditions show that the scheme is stable for CFL-numbers up to 1.3, and demonstrate a significant improvement in efficiency over the second order accurate method. A particularly difficult test problem is to propagate Rayleigh surface waves in an elastic half-space where the ratio between the compressional and shear wave speeds is large, i.e., an almost incompressible material. We demonstrate superior performance of the new scheme for this type of waves.

The new discretization of $(\mu(x)u_x)_x$ has general applicability, and will enable stable fourth order accurate approximations of other partial differential equations, as well as the elastic wave equation.

[1]This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC 52-07NA27344. This is contribution LLNL-ABS-493136.