A New Framework for Tracking Multiphase Physics: Foams, Membranes and Cells

(Frontiers in Computation: New Methods for Complex Mechanics, Advanced Materials, Interfaces, and Stochastics)

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Abstract

Many scientific and engineering problems are characterized by a large number of different regions touching in many different configurations, and whose interaction depends on complex physics. Examples include the motions of foams, crystal grain growth, and multicellular structures in man-made and biological materials, as well as mathematical and computational problems, such as geometric motion, domain decomposition and surface area minimization problems.

It is challenging to produce consistent and well-posed mathematical models that accurately model the hydrodynamic, elastic, diffusive, and transport processes that often characterize such motions. Building robust numerical schemes is equally daunting, especially in the presence of multi-junction points (triple points, quadruple points, etc.) in 2D and the analogous structures in 3D, including triple lines where multiple surfaces meet, etc.

In this talk, we will introduce a new set of computational methodologies to handle these problems. Our methods have several virtues. They use a single function on a fixed Eulerian mesh for an entire multiphase system, regardless of the number of phases, work in 2D/3D, and contain a real physical time that couples naturally to physics. The formulation automatically deals with the evolution of triple points/lines and topological change in the multiphase system, allowing phases to disappear and be created. The methods are first order accurate at triple points/lines, and arbitrarily high order away from these degeneracies. Finally, the methods have a computational complexity dependent only on the length of the interface.

Figure 1a shows the motion of a large number of connected phases undergoing curvature flow in which each phase maintains its own area while moving to minimize the total perimeter. Figure 1b/c shows multi-phase interactions under a large shear incompressible, variable viscosity Navier-Stokes flow computed with and without permeability. Our methods are applicable across a range of multi-scale/multi-physics problems. We shall discuss them in detail, with applications to industrial foam drainage and acini stability in biological cell membranes.

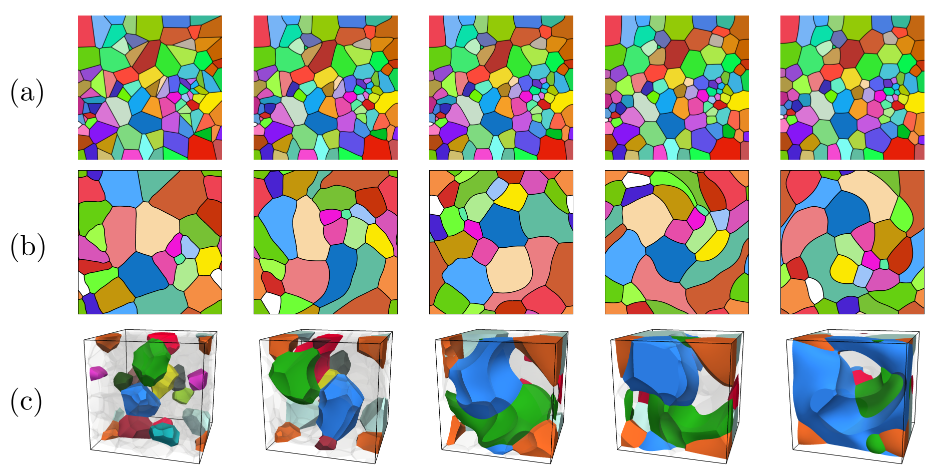

Figure 1: In all cases, time advances from left to right. (a) Curvature flow with area conservation on 100 initial random phases. (b) Navier-Stokes fluid flow simulation with an external agitator force and no permeability. (c) Navier-Stokes 3D fluid flow with surface tension and permeability (subset of phases shown).