Multilevel preconditioners for indefinite linear systems

Project title: Robust parallel iterative solvers for linear and least-squares systems

Yousef Saad
University of Minnesota
Dept. of Computer Science and Engineering
200 Union St. SE
Minneapolis, MN 55455

Abstract

When solving very large linear systems of equations one is faced with a confluence of difficulties which may render the task quite challenging for realistic problems. These problems tend to become intrinsically harder to solve by iterative methods by the nature of the physical application (e.g., Helmholtz or Maxwell, multiphysics, irregular problems). Also, because of their larger sizes (fine 3-D simulations) they also tend to be very ill-conditioned. Adding to this picture the complexity of a good parallel implementation for irregular problems, one often ends up with iterative methods that often fail or performs poorly.

The emphasis of this project is on improving robustness of iterative solvers, especially in highly parallel environments. To highlight a few recent contributions, we will mention some work related to solving challenging Helmholtz systems with preconditioned Krylov methods. We adapted some some techniques developed by others from a Partial Differential Equation (PDE) viewpoint into a purely algebraic viewpoint. The main method consists of perturbing the diagonal of a matrix by complex shifts before performing the ILU or ARMS factorization. The contribution here consists in determining how to shift the matrix in relation to the dropping strategy. The resulting preconditioner performs surprisingly well, both in terms of robustness and efficiency, for Helmholtz-related problems. More recently, we adapted these strategies for the so-called Modified ILUs (or diagonal compensation ILUs). Another highlight is our recent work on reordering based on *coarsening strategies*. Our initial goal was to define multilevel ILUs (based on our work on Algebraic Recursive Multilevel Solver (ARMS)) by exploiting general weighted graph coarsening. While this still remains our goal, we found that the method works remarkably well as a reordering for ILU. This work is continuing. Finally, we should also highlight our work on sparse matrix techniques for GPGPUS. Here, we have tried to answer the question: Can GPUs can be vastly superior to standard CPUs (with say a few cores) when employing *a complete* sparse iterative linear system solver. The answer is really mixed. It is possible to outperform CPUs by a good factor by using techniques like polynomial preconditiong (yes, they are back!) – but then the margin is somewhat limited, and possibly system specific, and we expect other serious issues to emerge for very large problems. Overall, our conclusion is that more is to be done on the hardware side.

In terms of software, we have currently three packages posted in the PI's web-site, all three of which are in big part supported by this effort: ARMS, ITSOL, and (new) CUDA_ITSOL. All are posted in `http://www-users.cs.umn.edu/~saad/software/`.

This is a joint effort with Masha Sosonkina (Ames Lab., Iowa).