Petascale Polynomials and a Real Analogue of Smale's 17th Problem

ASCR: Topology for Statistical Modeling of Petascale Data

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Abstract

Smale's 17th Problem asks whether a solution of n complex polynomial equations in n unknowns can be found approximately, on the average, in polynomial time with a uniform algorithm. Recent work of Beltran, Bürgisser, Cucker, and Pardo, has brought us closer to a positive answer. However, the analogous question for real solutions is open, and of even greater practical importance.

We discuss a method, along with a preliminary implementation in `Sage`, that significantly speeds up the approximation of *real* solutions of systems of polynomial equations. To clarify our speed-up, suppose f is a polynomial in n variables with real coefficients, degree d, and exactly $n+k$ monomial terms. So far, all algorithms for deciding whether f has a real zero have complexity polynomial in d^n, even in the average-case.

Suppose k is fixed but otherwise arbitrary. Our techniques yield an algorithm that, for certain probability distributions, counts exactly the number of connected components of the real zero set of f in average-case time polynomial in $n + \log d$. Our techniques also yield an approach to the original Smale's 17th Problem via polyhedral homotopy. Our key advance is an efficient method to determine which connected component of a particular $\mathcal{A}$-discriminant complement contains f. We present numerous examples illustrating our techniques and assume no background in algebraic geometry.

This is joint work with Philippe Pébay, Korben Rusek, and David Thompson.