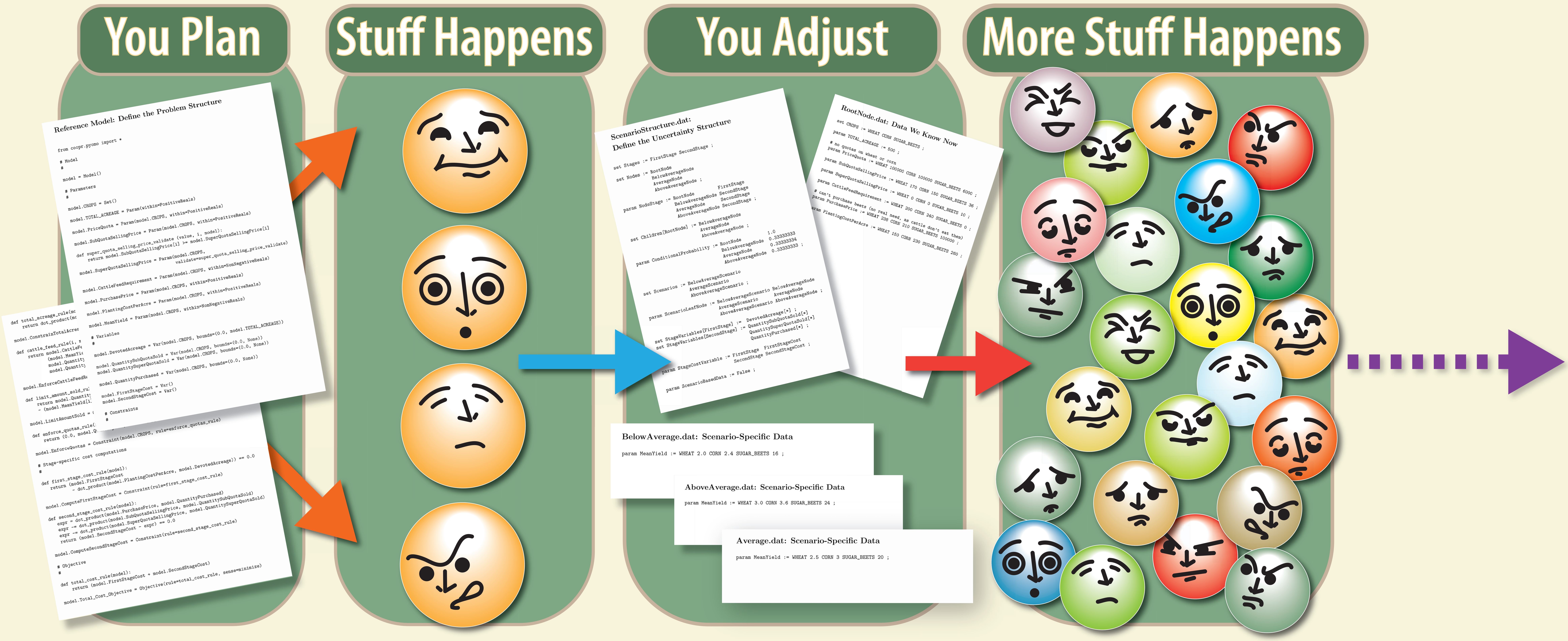
Hedging Against Uncertainty:
A Modeling Language and Solver Library



PYOMO

PySP: Stochastic Programming in Python

COOPR

Multi-Stage Planning for Uncertain Environments

- Explicitly capture recourse
- Uncertainty modeling framework
- Integrated solver strategies

What We Do:

- Mixed decision variables
  - Continuous
  - Integer/Binary
- General multi-stage
- Stochastic programming
  - Expected value
  - Conditional Value-at-Risk
  - Scenario selection
- Cost confidence intervals

How We Do It:

- Deterministic equivalent
- Scenario-based decomposition
  - Progressive Hedging
  - Customizable accelerators
- Algebraic modeling via Pyomo
- SMP and cluster parallelism
- Integrated high-level language support
- Multi-platform, unrestrictive license
- Open source, actively supported by Sandia
- Co-Managed by Sandia and COIN-OR

UC DAVIS GRADUATE SCHOOL of MANAGEMENT *Ideas into Action*

TO LEARN MORE VISIT > https://software.sandia.gov/trac/coopr/wiki/PySP

Sandia National Laboratories

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Progressive Hedging: Basic Pseudo-Code

1. $k := 0$

2. For all $s \in \mathcal{S}$, $x_s^{(k)} := \text{argmin}_x (c \cdot x + f_s \cdot y_s) : (x, y_s) \in \mathcal{Q}_s$

3. $\bar{x}^k := (\sum_{s \in \mathcal{S}} p_s\, d_s x_s^{(k)}) / \sum_{s \in \mathcal{S}} p_s\, d_s$

4. For all $s \in \mathcal{S}$, $w_s^{(k)} := \rho(x_s^{(k)} - \bar{x}^{(k)})$

5. $k := k + 1$

6. For all $s \in \mathcal{S}$, $x_s^{(k)} := \text{argmin}_x (c \cdot x + w_s^{(k-1)} x + \rho/2 \left\| x - \bar{x}^{(k-1)} \right\|^2 + f_s \cdot y_s)$
$: (x, y_s) \in \mathcal{Q}_s$

7. $\bar{x}^{(k)} := (\sum_{s \in \mathcal{S}} p_s\, d_s x_s^{(k)}) / \sum_{s \in \mathcal{S}} p_s\, d_s$

8. For all $s \in \mathcal{S}$, $w_s^{(k)} := w_s^{(k-1)} + \rho \left(x_s^{(k)} - \bar{x}^{(k)} \right)$

9. $g^{(k)} := \frac{(1-\alpha)|\mathcal{S}|}{\sum_{s \in \mathcal{S}} p_s\, d_s} \sum_{s \in \mathcal{S}} \left\| x^{(k)} - \bar{x}^{(k)} \right\|$

10. If $g^{(k)} < \epsilon$, then go to step 5. Otherwise, terminate.

Rockafellar and Wets (1991)

Parallelization and Scenario-Based Decomposition

- Progressive Hedging is “trivially” parallelizable
  - Each batch of sub-problem solves is independent
  - So what’s the big deal?

- Maintaining parallel efficiency is a major issue in any practical implementation

- Key problem drivers
  - High variability in sub-problem solve times
  - Sub-problem solves too fast => communication dominates

- Key solution strategies
  - Relaxing barrier synchronization after each batch of sub-problem solves
  - Scenario “bundling” to increase sub-problem difficulty and to accelerate PH convergence

Asynchronous Sub-Problem Solves in PH

- In the case of mixed-integer optimization problems, variability of sub-problem solve times can be considerable
  - Observations "in the wild" vary over 4 or more orders of magnitude

- The presence of such dramatic variability clearly destroys any potential benefit of parallelism in PH

- Our solution
  - Relax the barrier synchronization, allow for asynchronous solves
  - Retains PH convergence properties, as long as sub-problem solves for each scenario periodically report back

- Challenges and Results
  - Significant interference with mixed-integer acceleration mechanisms
  - Slows PH convergence, but (empirically) only by a constant factor

Scenario Bundling in PH

- General idea
  - Cluster scenarios using some similarity (or dis-similarity) metric
  - Forming miniature "extensive forms"
- Benefits
  - Increases sub-problem solve times, dropping comm:compute ratio
  - (Often) dramatic accelerations in PH convergence
- Research questions
  - Do we bundle based on maximal similarity or maximal differences?
  - How to handle bundling in multi-stage scenario trees?
- Preliminary results
  - Even pairing of scenarios randomly yields very large reductions in the number of PH iterations required for convergence

Driver Applications for Asynch PH and Bundling

- Stochastic Unit Commitment
  - Two and multi-stage stochastic mixed-integer
- Transmission and Generation Expansion
  - Two and multi-stage stochastic mixed-integer
- Parameter estimation
  - Childhood disease models (SIR)
  - Two and multi-stage stochastic non-linear
- Network design
  - Academic, but very difficult (two-stage mixed-integer stochastic programs)
- Forestry management
  - Multi-stage mixed-integer; determining harvest schedule

Software and Contact Information

- All of these techniques are available in our PySP open-source software package
  - https://software.sandia.gov/trac/coopr/wiki/PySP
  - Distributed by Sandia and COIN-OR
  - Jointly developed and maintained by Sandia and UC Davis

- Asynchronous PH and bundling interfaces are currently supported in PySP
  - Alpha, but functional
  - The rest of PySP is rather stable

- Feel free to contact us!
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