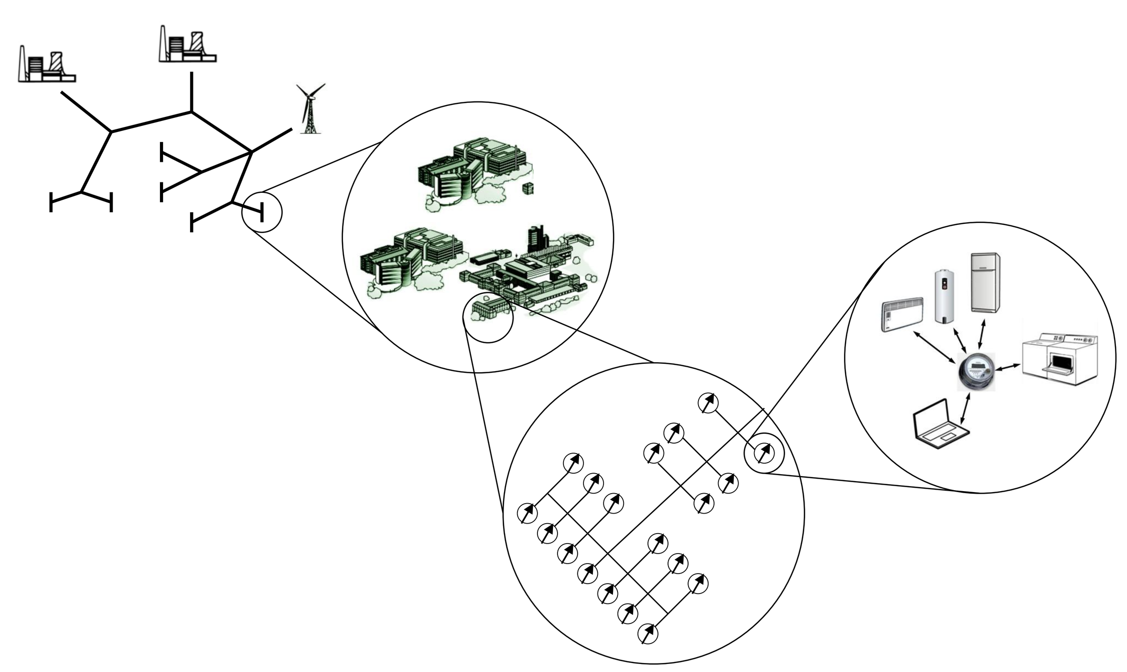
Random transitions and eigenvalues for modeling demand and stability for the Smart Grid

Hadi Meidani, Roger Ghanem
Viterbi School of Engineering, University of Southern California

I. Motivation

Challenges

- Complex system of interacting systems
- Uncertainty due to intermittent energy resources
- Uncertainty due to price-dependent electricity demand
- Multiple temporal and spatial scales



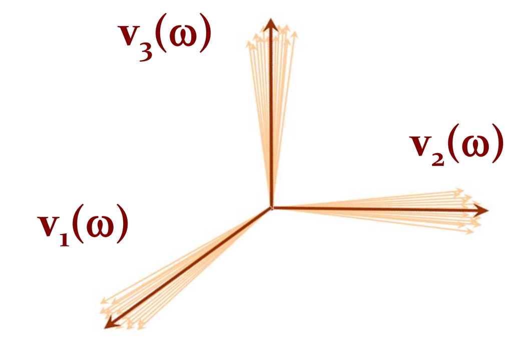
Developed Methodologies

1. Random eigenvalue analysis for stability analysis of interconnected systems under uncertainty.
2. Markov chains with random transition matrices for demand modeling.
3. Cooperation in networked multi-agent systems in random environment for uncertainty quantification of agreement in distribution network.

II. Random eigenvalue analysis

Introduction

- An efficient solution algorithm which extends Subspace Iteration method to random eigenvalue problems.
- Polynomial Chaos representation of random eigenpairs.
- Uncertainty quantification for dominant subspace.
- Sensitivity indices w.r.t. uncertainty sources are readily computable.



$$A(\omega)\boldsymbol{v}_m(\omega) = \lambda_m(\omega)\boldsymbol{v}_m(\omega)$$

Algorithm Deterministic subspace iteration

Step 1 Choose an initial set of m normalized vectors $V^{(0)} = [\boldsymbol{v}_1^{(0)}, \cdots, \boldsymbol{v}_m^{(0)}]$

$k = 0$

while $\delta > \epsilon_{tol}$ **do**

  $k = k + 1$

  Step 2 $Y = AV^{(k)}$

  Step 3 Compute the QR factorization, $Y = QR$, and set $V^{(k)} \leftarrow Q$

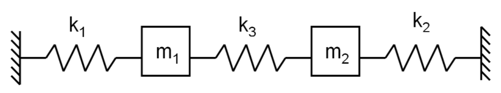
  $\delta = \sum_i \|\boldsymbol{v}_i^{(k)} - \boldsymbol{v}_i^{(k-1)}\|$

end while

Step 4 $\lambda_i = (\boldsymbol{v}_i^{(k)})^T A(\boldsymbol{v}_i^{(k)}) \qquad i = 1, \cdots, m$

Illustration I

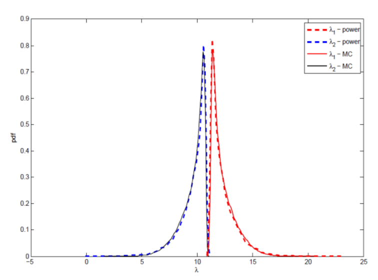
- 2DOF mass-spring
- Randomness in the spring 1:



$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$K(\xi) = \begin{bmatrix} 11 + 2\xi & -1 \\ -1 & 11 \end{bmatrix}$$

- Exact eigenvalue solution:

$$\lambda_1(\xi) = 11 + \xi + \sqrt{\xi^2 + 1}$$
$$\lambda_2(\xi) = 11 + \xi - \sqrt{\xi^2 + 1}$$


Convergence in distribution

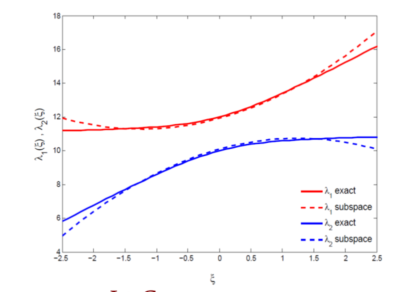
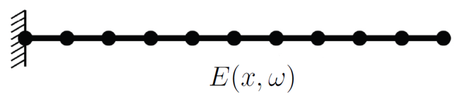
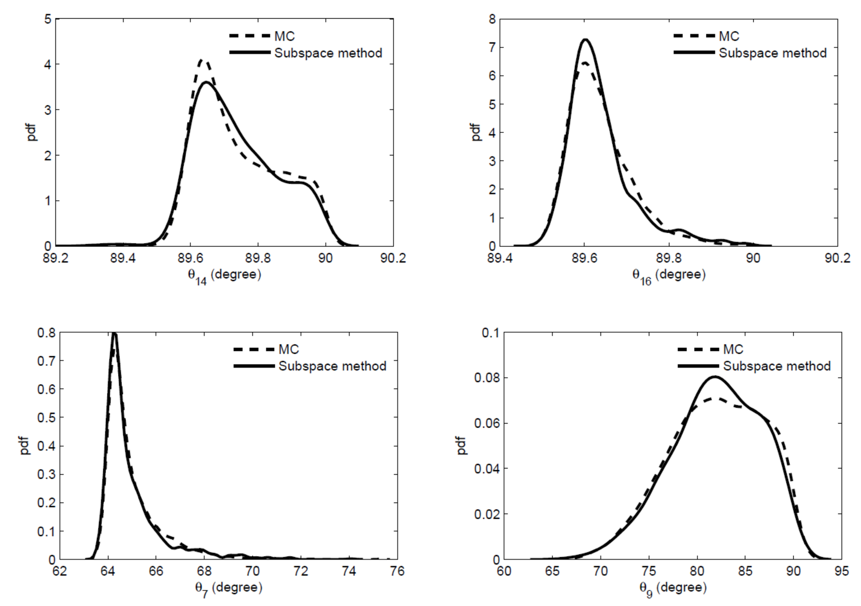

L² Convergence

Illustration II

- Elastic beam with random Young's modulus
- 10 element discretization and Polynomial representation for the 20x20 stiffness matrix – 2nd order with 3 uncertainty sources
- Calculation of the first 4 eigenpairs



$E(x, \omega)$



Figures: (top) representative pdf's of the phase angles of the first eigenvector; (bottom) representative pdf's of the phase angles of the 4th eigenvector

III. Random Markov transition matrices

Introduction

- Uncertainties due to measurement noise, missing information and inherent variability of transition rates result in uncertainty in the estimation procedure.
- Maximum Entropy formulation for random transition matrix given mean values and standard deviations obtained from observation.

Mathematical Setting

- Admissible sets:

$$\Pi = \{\pi \in \mathbb{R}^{1\times n} \mid \sum_{i=1}^{n} \pi_i = 1, w_i \in [0,1]\}$$
$$\mathcal{P} = \{P \in \mathbb{R}^{n\times n} \mid P\mathbf{1} = \mathbf{1} \ \ \forall i \in \mathcal{S}, p_{ij} \geq 0\}$$

Maximum entropy density estimation

- Mean values calculated based on transition records:

$$\bar{p}_{ij} = \frac{c_{ij}}{\sum_j c_{ij}}$$

- Independent estimation of joint pdf's for each row:

$$P(\omega) = \begin{bmatrix} \vdots \\ \boldsymbol{r}(\omega) \\ \vdots \end{bmatrix}$$

- Lower dimensional random vector:

$$\boldsymbol{r}'(\omega) = [r_1(\omega), \cdots, r_{n-1}(\omega)]$$

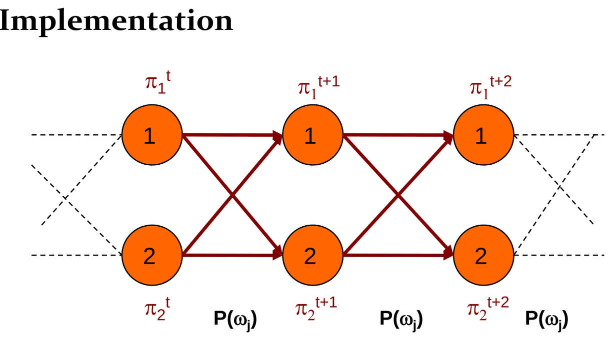
- MaxEnt joint pdf:

$$f_{r'}^*(\boldsymbol{r}') = e^{(\mu-1)} \exp(\sum_{i=1}^{n-1} \lambda_i r_i + \sum_{i=1}^{n-1} \eta_i r_i^2) \mathbb{1}_{V'}(\boldsymbol{r}')$$

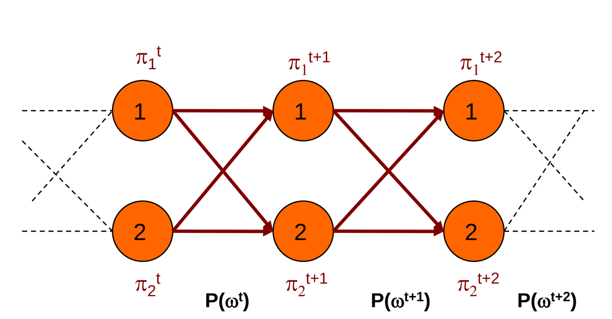
- Support:

$$V' = \left\{ \boldsymbol{r}' \subset \mathbb{R}^{n-1} \mid \sum_{i=1}^{n-1} r_i' \leq 1; \ \boldsymbol{r}' \geq 0 \right\}$$

Implementation



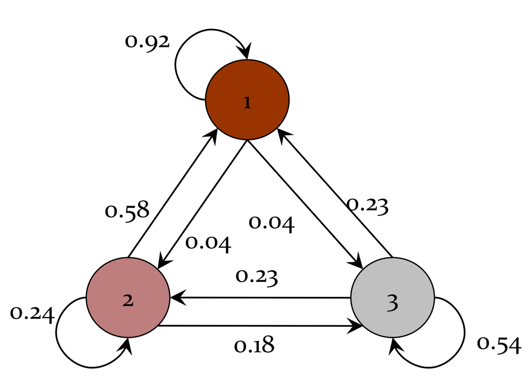
Time-homogeneous implementation



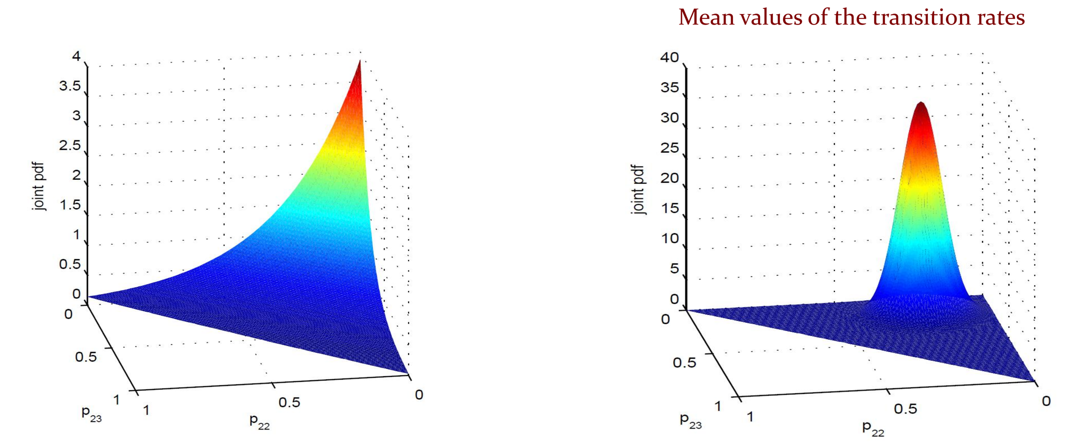
Time-heterogeneous implementation

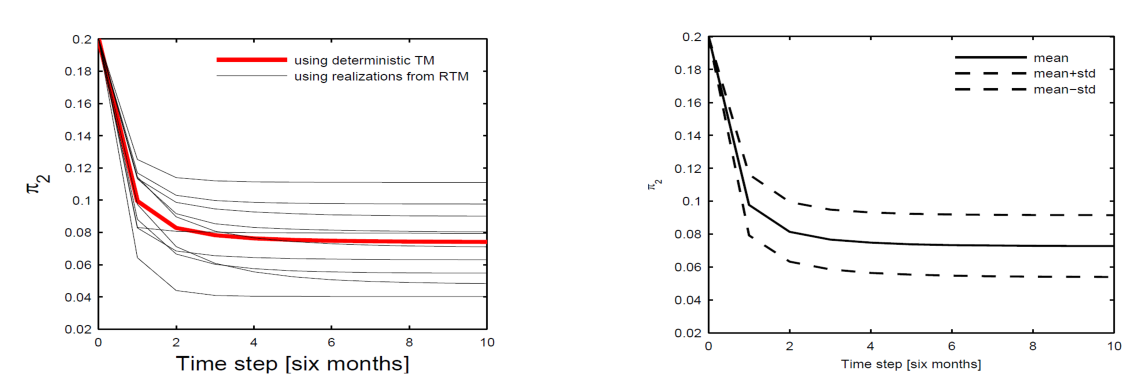
Illustration

- HIV progression; the 3 states are defined based on the cell counts.
- Mean values and 10% c.o.v. are used.

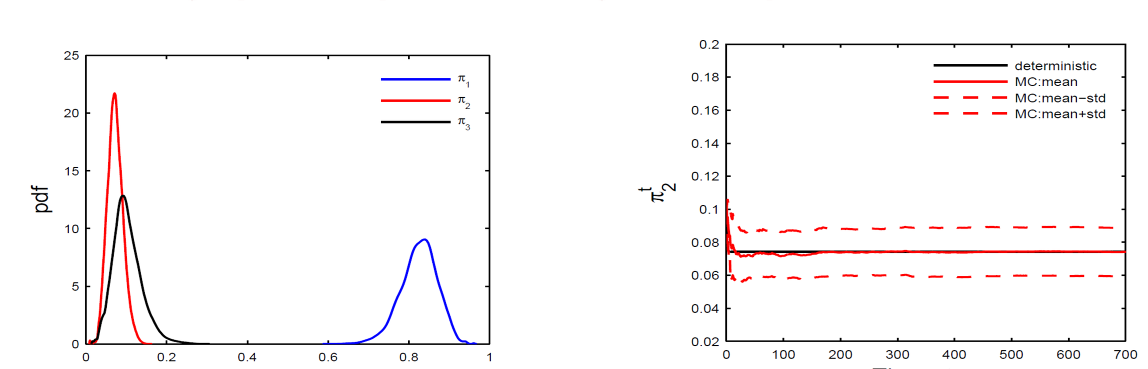


Mean values of the transition rates


Joint pdf of the 2 random transition rates: p_{22}, p_{23} – (left) only mean values used (right) mean values and variance used



Time response of state distribution π_2 – (left) comparison between deterministic and realized paths, (right) probabilistic path calculated using sufficient samples from MaxEnt RTM



(Left) stationary state distributions in time homogeneous implementation, (right) convergence in the time-heterogeneous implementation

IX. Synchronization in random networked systems

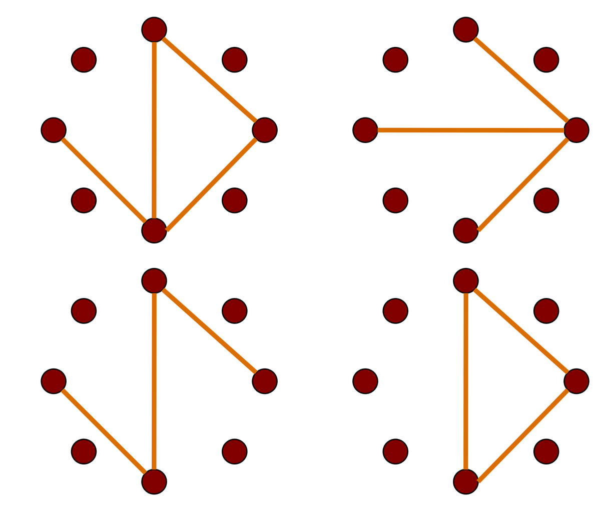
Introduction

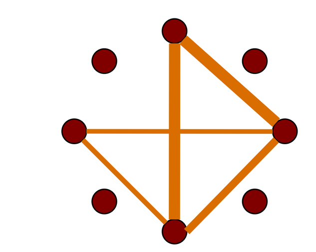
- Sources of uncertainty: limited communication, non-deterministic interactions, unobserved links, delays.
- Available data: status of interactions over a time period resulting in mean values and standard deviations.
- Fuzzy treatment of interconnection through a MaxEnt random matrix formulation.
- With adjacency matrix A, Graph Laplacian L, and time step size δ_t

$$x_i^{k+1} = x_i^k + \delta_t \sum_j a_{ij}(x_i^k - x_j^k)$$

- Markov chain representation under uncertainty:

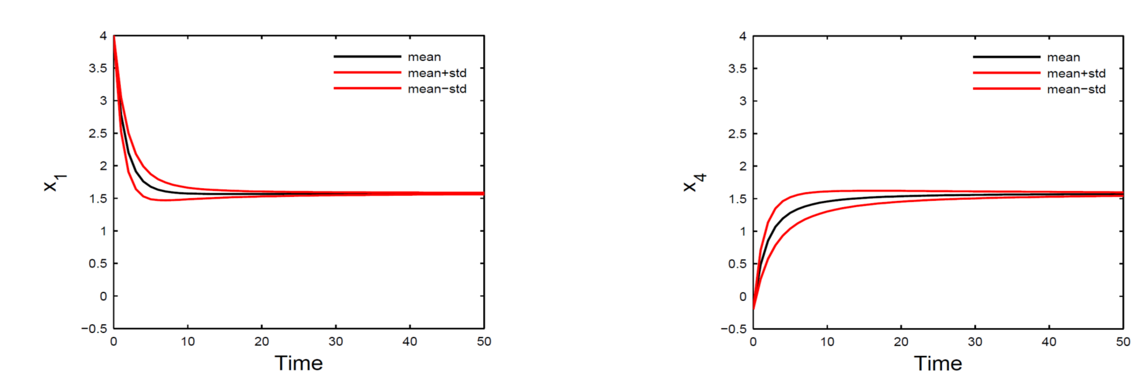
$$\boldsymbol{x}^{k+1}(\omega) = (I - \delta_t L(\omega))\, \boldsymbol{x}^k(\omega)$$
$$= P(\omega)\, \boldsymbol{x}^k(\omega)$$


(Top) Non-weighted graphs; 4 samples from links
(Bottom) weighted graph; Probabilistic representation of links



Illustration

- Quantification of the scatter (our ignorance) about the consensus under uncertainties.
- Estimation of the time after which the agreement is achieved within an acceptable discrepancy range.



Time response of the random value of agents 1 and 4

Ackowledgment

This research is part of the project "Analysis and Reduction of Complex Networks Under Uncertainty" in collaboration with Youssef Marzouk (MIT), Omar Knio (Duke), Bert Debusschere and Habib Najm (Sandia) and is sponsored by a DOE-ASCR grant.