Energy Aware Scheduling

Algorithm

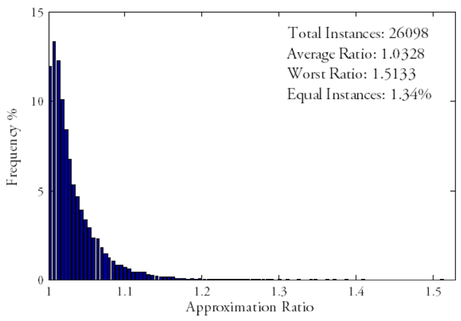
SCHEDULE BY α-INTERVALS AND α-SPEEDS (SAIAS)

Inputs: set of jobs, $\alpha \in (0,1)$, $\epsilon > 0$, set of speeds $\mathbf{S} = \{\sigma_1, \ldots, \sigma_m\}$.

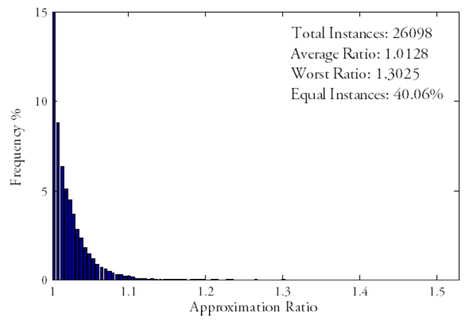
1 Compute an optimal solution $\bar{\mathbf{x}}$ to the linear relaxation of the modified interval-indexed IP.
2 Compute the α-intervals τ^α and the sets J_t.
3 Compute an order Π^α that has the sets J_t ordered in non-decreasing values of t and the jobs within each set in a manner consistent with the precedence constraints.
4 Compute the α-speeds $\mathbf{s}^\alpha$
5 Round down each s_i^α to the nearest speed in $\mathbf{S}$ and run job i at this rounded speed, $\bar{s}_i^\alpha$.
6 Set the i-th job to start at time $\max\{r_{\pi(i)}, \bar{C}^\alpha_{\pi(i-1)}\}$, where $\bar{C}^\alpha_{\pi(i-1)}$ is the completion time of the previous job using the rounded α-speeds, and $\bar{C}^\alpha_{\pi(0)} = 0$.
7 **SAIAS-H**: re-optimize speeds given the order Π^α.
8 **return** speeds $\bar{\mathbf{s}}^\alpha$ and schedule $(\Pi^\alpha, \bar{\mathbf{C}}^\alpha)$.

Experimental Results

Performance (n = 7)

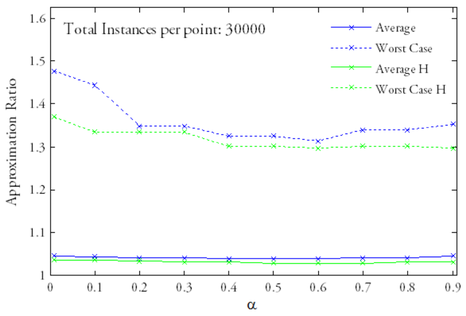


SAIAS with release dates

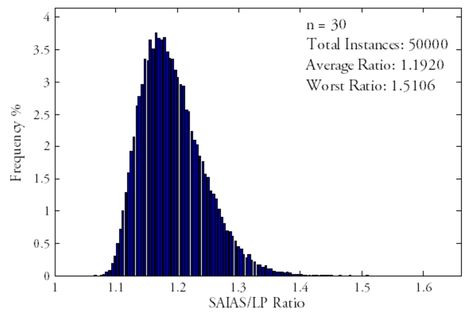


Heuristic Improvement

Sensitivity Experimental Results

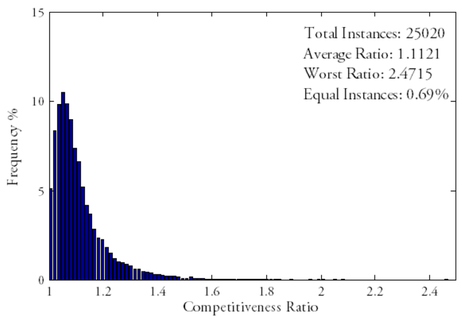


Sensitivity o α (n=7)

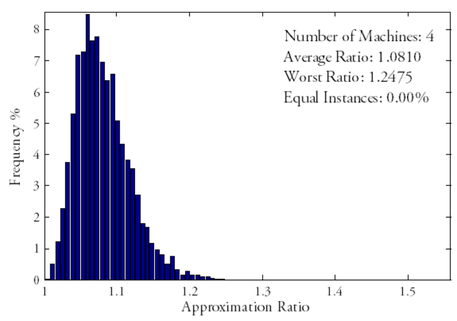


LP solution vs. SAIAS for large Instances (n=30)

Extensions

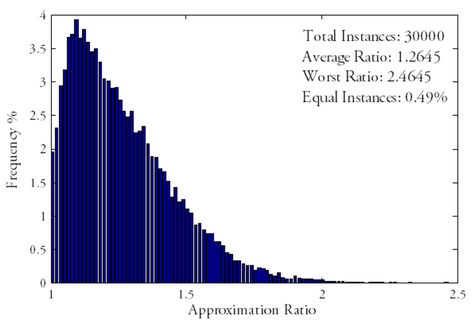


Online (n=11)

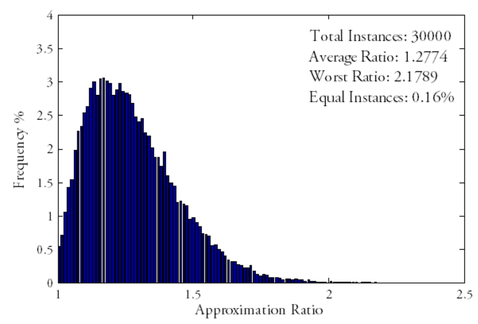


Multiple Machines (m=4, n=15)

SAIAS-T without Speed-up – Modification for the Tardiness Case



No release dates (n=7)



With release dates (n=7)

Interval and Speed Indexed IP

$$\min_{\mathbf{x}} \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{t=1}^{T} \left(E_i(\sigma_j) + w_i\tau_{t-1}\right) x_{ijt}$$

s.t.

$$\sum_{j=1}^{m}\sum_{t=1}^{T} x_{ijt} = 1, \ \forall i, \tag{1}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{u=1}^{t} \frac{\rho_i}{\sigma_j} x_{iju} \le \tau_t, \ \forall t, \tag{2}$$

$$x_{ijt} = 0, \quad \text{if } \tau_t < r_i + \frac{\rho_i}{\sigma_j}, \ \forall i, \forall j, \forall t, \tag{3}$$

$$i_1 \prec i_2 \Rightarrow \sum_{j=1}^{m}\sum_{u=1}^{t} x_{i_1ju} \ge \sum_{j=1}^{m}\sum_{u=1}^{t} x_{i_2ju}, \ \forall t. \tag{4}$$

Theoretical Results

Theorem 1. *The SAIAS algorithm with $\alpha = \frac{1}{2}$ is a $4(1+\epsilon)(1+\delta)$-approximation algorithm for the $1 \mid prec \mid \sum E_i(s_i) + \sum w_iC_i$ problem, and with $\alpha = \sqrt{2}-1$ is a $(3+2\sqrt{2})(1+\epsilon)(1+\delta)$-approximation algorithm for the $1 \mid r_i, prec \mid \sum E_i(s_i) + \sum w_iC_i$ problem, for all general non-negative energy cost functions $E_i(s_i)$.*

Problem	Algorithm	Average Ratio	Worst Ratio	Optimal Instances
$1 \mid prec \mid \sum E_i(s_i) + \sum w_iC_i$	SAIAS	1.0077	1.1437	28.39%
	SAIAS-H	1.0045	1.0997	48.07%
$1 \mid r_i, prec \mid \sum E_i(s_i) + \sum w_iC_i$	SAIAS	1.0328	1.5133	1.34%
	SAIAS-H	1.0128	1.3025	40.06%
Bimodal ρ=3	SAIAS	1.0067	1.1055	33.59%
	SAIAS-H	1.0038	1.0755	54.12%
Bimodal ρ=5	SAIAS	1.0086	1.1019	25.88%
	SAIAS-H	1.0046	1.0943	49.22%
Bimodal ρ=10	SAIAS	1.0101	1.1263	16.92%
	SAIAS-H	1.0037	1.1158	50.90%
Bimodal ρ=15	SAIAS	1.0101	1.1469	14.70%
	SAIAS-H	1.0030	1.1246	53.73%
Polynomial Energy	SAIAS	1.0360	1.9785	40.45%
Online, large r_i	SAIAS-H Online	1.0150	1.3070	21.59%
Online, worst r_i	SAIAS-H Online	1.1121	2.4715	0.69%
$P \mid\mid \sum E_i(s_i) + \sum w_iC_i$	SAIAS-P	1.0810	1.2475	0%
$1 \mid prec \mid \sum E_i(s_i) + \sum w_iT_i$	SAIAS-T	3.5464	22.7207	0%
	SAIAS-T NS	1.2645	2.4645	0.49%
$1 \mid r_i, prec \mid \sum E_i(s_i) + \sum w_iT_i$	SAIAS-T	2.8929	16.2307	0%
	SAIAS-T NS	1.2774	2.1789	0.16%

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