Numerical Analysis of Multiple Time Scale Dynamical Systems

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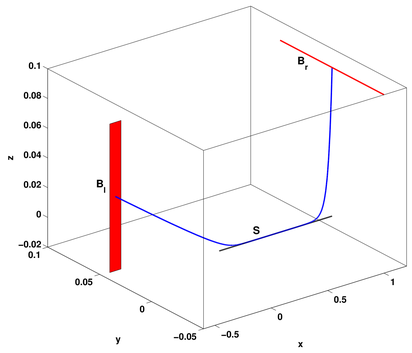
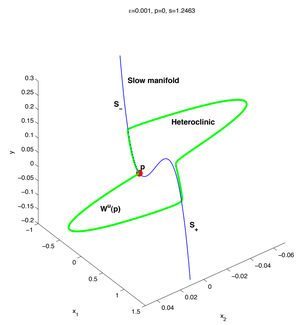
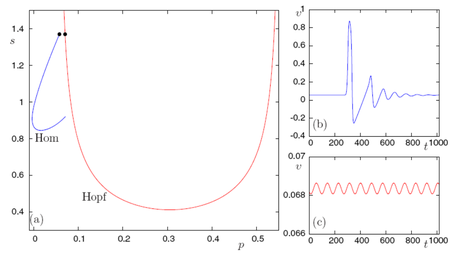
Recent Highlights



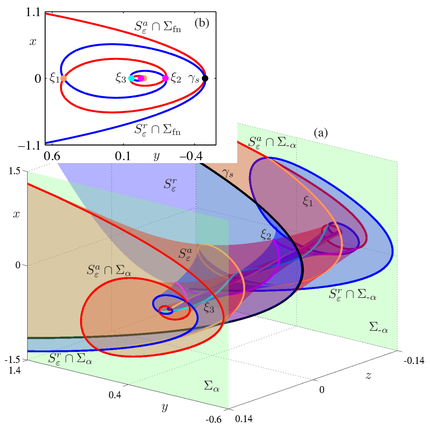
Algorithms for slow manifolds of saddle-type



Traveling waves of FitzHugh-Nagumo equation



Invariant manifold tangencies in CU bifurcations



Mechanisms for mixed mode oscillations

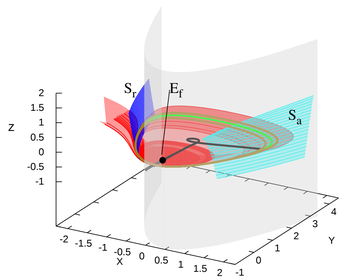
New Results

Singular Hopf Bifurcation
Philipp Meerkamp

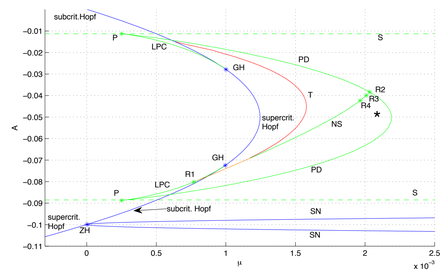
- Occurs when an equilibrium crosses a fold of a slow manifold
- Normal form in systems with two slow variables: three "secondary" parameters

$$\dot{x} = (y - x^2)/\varepsilon$$
$$\dot{y} = z - x$$
$$\dot{z} = -\mu - ax - by - cz$$

- Comprehensive analysis of bifurcations in this normal form



(mu,a,b,c) = (0.0022,-0.05,0.001,0.1)

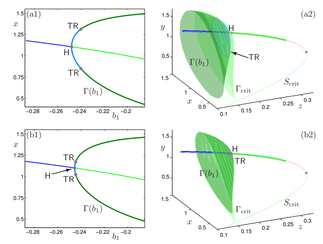


(b,c) = (0.001,0.1)

- Curve T are parameters where repelling slow manifold is tangent to unstable manifold of equilibrium
- T curve bounds region of mixed mode oscillations in systems with "global return"
- Underway: Computer verified proof of these tangencies

Normal Forms of Dynamic Hopf Bifurcation
Hinke Osinga

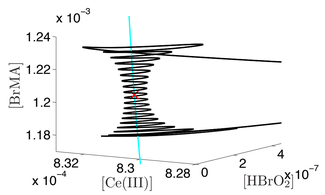
- Dynamic Hopf bifurcation is Hopf bifurcation in *fast* variables of slow-fast system
- First Lyapunov coefficient determines when bifurcation is subcritical or supercritical
- Result 1: First Lyapunov coefficient may change sign at singular limit
- Result 2: When this happens, there is a nearby torus bifurcation

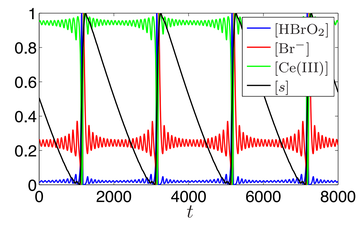


- Diagram: Hindmarsh-Rose model of neuronal bursting for two different values of epsilon. The bifurcation is supercritical but quickly undergoes torus bifurcation.
- Theoretical analysis introduces analytically tractable example

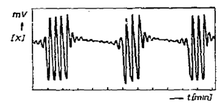
Models of BZ Reaction
Chris Scheper

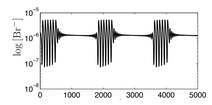
- Belousov-Zhabotinsky reaction has mixed mode oscillations in stirred tank
- Many differential equations models proposed and studied (from 1972) but none have been shown to match extensive experimental data
- Extensive analysis of four dimensional model proposed by Gyorgyi and Field as reduction of more detailed mass action kinetics
- No explicit separation of time scales, but use multiple time scale methods





- Two separated regions of mixed mode oscillations
- Dynamic Hopf bifurcation in region of low flow rate MMOs (shown)
- Singular Hopf bifurcation in region of high flow rate MMOs

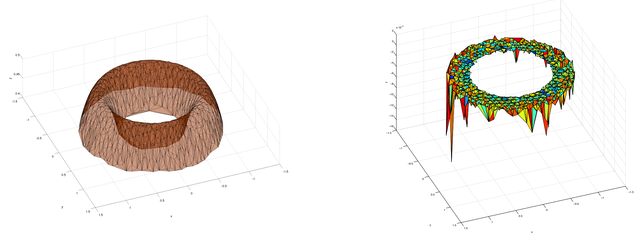




- Comparison of model results with published time series (Marek and Svobodoba)

Smooth Multivariate Interpolation

- Interpolation is mature subject for functions of one variable: much less is known for functions of more variables
- Subdivision algorithms of computer graphics give C^1 or C^2 smoothness with complex rules at exceptional nodes of mesh
- Whitney extension theorem and recent improvements (Fefferman et al.) provide theoretical context
- Goal: Simple, infinitely differentiable interpolation with good accuracy for points on smooth surface



- Algorithm: partition of unity with carefully selected cover based upon Delaunay triangulation of mesh
- Initial tests are encouraging: figure is part of torus and the approximation errors